Autumn Scheme of Learning

Year 6

#MathsEveryoneCan

2020-21

White Rose Maths

New for 2020/21

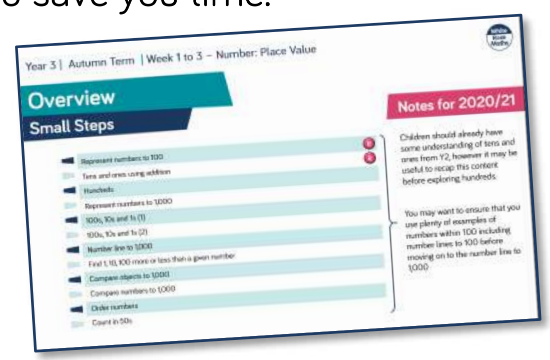
2020 will go down in history. The world has changed for all of us.

We want to do as much as we can to support children, teachers, parents and carers in these very uncertain times.

We have amended our schemes for 2020/21 to:

- ★ highlight key teaching points
- ★ recap essential content that children may have forgotten
- ★ flag any content that you might not have covered during the school closures period.

We hope these changes will add further value to the schemes and save you time.



Lesson-by-lesson overviews

We've always been reluctant to produce lesson-by-lesson overviews as every class is individual and has different needs. However, many of you have said that if blended learning becomes a key feature of school life next year, a weekly plan with linked content and videos could be really useful.

As always, we've listened! We've now produced a complete lesson-by-lesson overview for Y1 to Y9 that schools can use or adapt as they choose. Each lesson will be linked to a free-to-use home learning video, and for premium subscribers, a worksheet. This means that you can easily assign work to your class, whether they are working at home or in school.

Inevitably, this lesson-by-lesson structure won't suit everyone, but if it works for you, then please do make use of this resource as much as you wish.

Teaching for Mastery

These overviews are designed to support a mastery approach to teaching and learning and have been designed to support the aims and objectives of the new National Curriculum.

The overviews:

- have number at their heart. A large proportion of time is spent reinforcing number to build competency
- ensure teachers stay in the required key stage and support the ideal of depth before breadth.
- ensure students have the opportunity to stay together as they work through the schemes as a whole group
- provide plenty of opportunities to build reasoning and problem solving elements into the curriculum.

For more guidance on teaching for mastery, visit the NCETM website:

https://www.ncetm.org.uk/resources/47230

Concrete - Pictorial - Abstract

We believe that all children, when introduced to a new concept, should have the opportunity to build competency by taking this approach.

Concrete – children should have the opportunity to use concrete objects and manipulatives to help them understand what they are doing.

Pictorial – alongside this children should use pictorial representations. These representations can then be used to help reason and solve problems.

Abstract – both concrete and pictorial representations should support children's understanding of abstract methods.

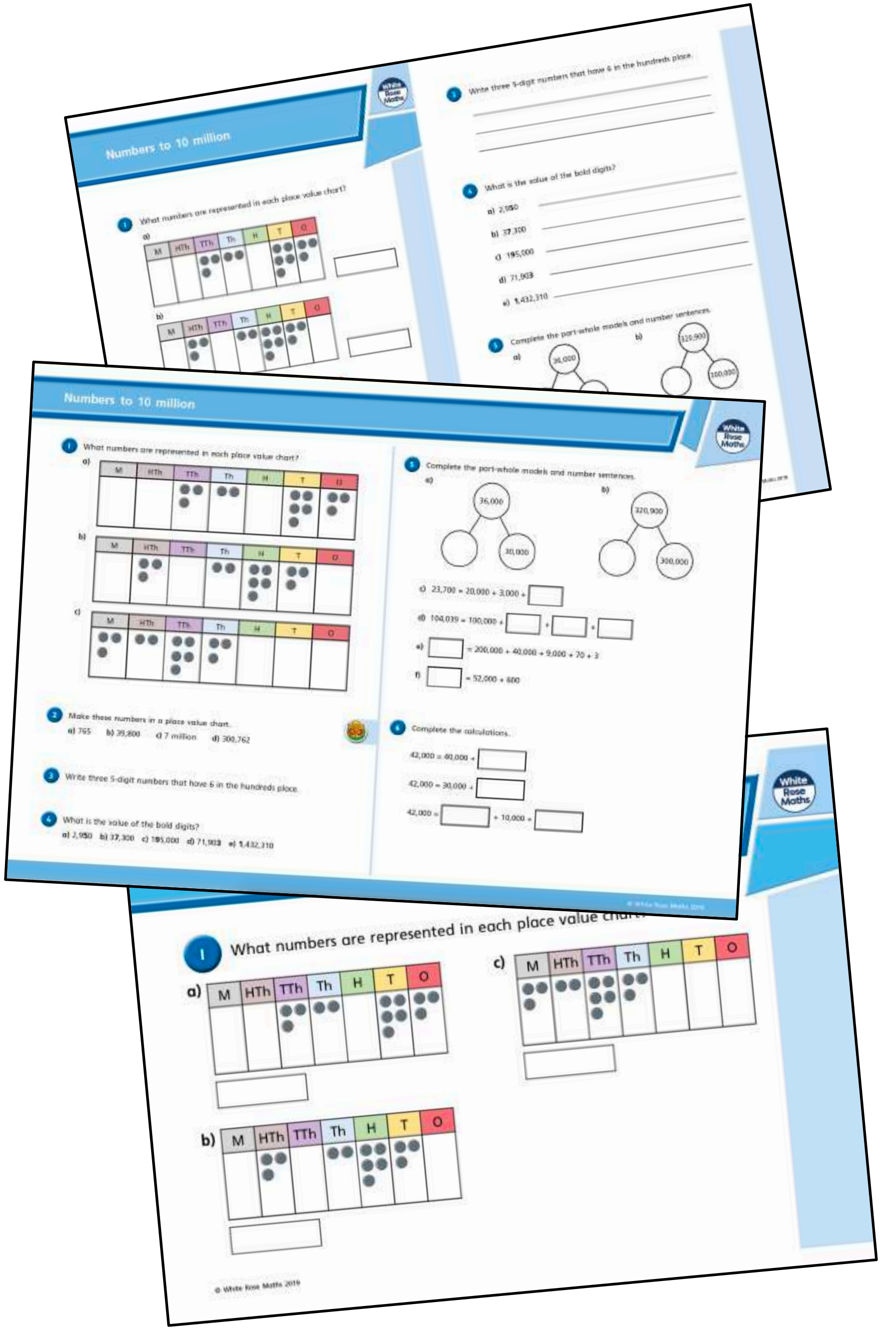
Need some CPD to develop this approach? Visit www.whiterosemaths.com for find a course right for you.

Supporting resources

We have produced supporting resources for every small step from Year 1 to Year 11.

The worksheets are provided in three different formats:

- Write on worksheet – ideal for children to use the ready made models, images and stem sentences.
- Display version – great for schools who want to cut down on photocopying.
- PowerPoint version – one question per slide. Perfect for whole class teaching or mixing questions to make your own bespoke lesson.

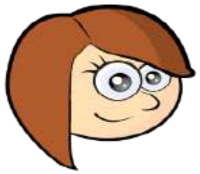


For more information visit our online training and resources centre resources.whiterosemaths.com or email us directly at support@whiterosemaths.com

Meet the Characters

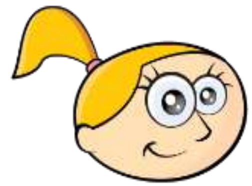
Children love to learn with characters and our team within the scheme will be sure to get them talking and reasoning about mathematical concepts and ideas. Who's your favourite?

Teddy

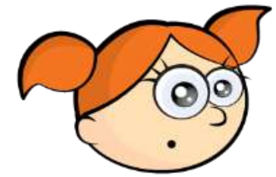


Rosie

Mo



Eva



Alex

Jack

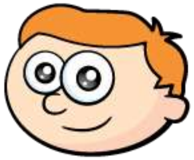
Whitney

Amir

Dora

Tommy

Dexter



Ron

Annie

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	Number: Place Value		Number: Addition, Subtraction, Multiplication and Division				Number: Fractions					Geometry: Position and Direction
Spring	Number: Decimals		Number: Percentages		Number: Algebra		Measurement: Converting Units	Measurement: Perimeter, Area and Volume		Number: Ratio		Statistics
Summer	Geometry: Properties of Shape			Consolidation or SATs preparation		Consolidation, investigations and preparations for KS3						

White Rose Maths

Autumn - Block 1

Place Value

Overview

Small Steps

- Numbers to 10,000 (R)
- Numbers to 100,000 (R)
- Numbers to a million (R)
- Numbers to ten million
- Compare and order any number
- Round numbers to 10, 100 and 1,000 (R)
- Round any number
- Negative numbers

Notes for 2020/21

Many children may struggle to work immediately with numbers to 10,000,000 so we are suggesting that this might build up from smaller numbers.

It's vital that children have that understanding/recap of place value to ensure they are going to be successful with later number work.

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Numbers to 10,000

Notes and Guidance

Children use concrete manipulatives and pictorial representations to recap representing numbers up to 10,000

Within this step, children must revise adding and subtracting 10, 100 and 1,000

They discuss what is happening to the place value columns, when carrying out each addition or subtraction.

Mathematical Talk

Can you show me 8,045 (any number) in three different ways?

Which representation is the odd one out? Explain your reasoning.

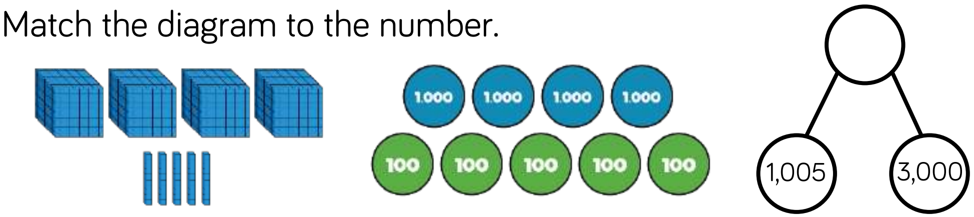
What number could the arrow be pointing to?

Which column(s) change when adding 10, 100, 1,000 to 2,506?

Varied Fluency

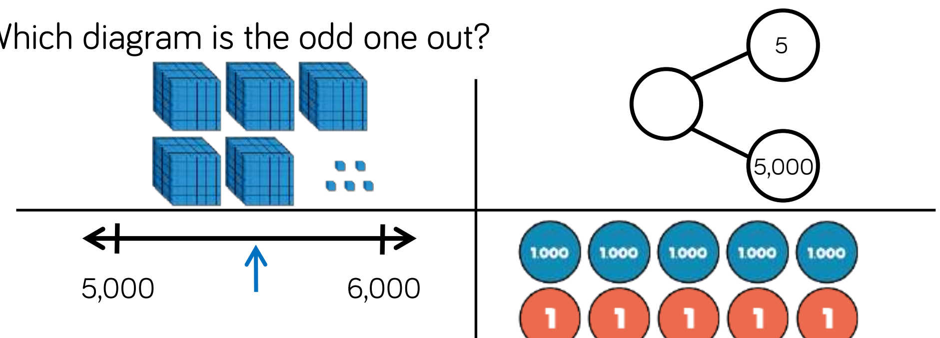
R

Match the diagram to the number.



4,005 | 4,500 | 4,050

Which diagram is the odd one out?



Complete the table.

	Add 10	Add 100	Add 1,000
2,506			
7,999			
		6,070	

Numbers to 10,000

Reasoning and Problem Solving

Dora has made five numbers, using the digits 1, 2, 3 and 4 She has changed each number into a letter. Her numbers are aabcd acdbc dcaba cdadc bdaab Here are three clues to work out her numbers: • The first number in her list is the greatest number. • The digits in the fourth number total 12 • The third number in the list is the smallest number.	44,213 43,123 13,424 31,413 21,442

Tommy says he can order the following numbers by only looking at the first three digits. 12,516 12,832 12,679 12,538 12,794 Is he correct? Explain your answer.	He is incorrect because two of the numbers start with twelve thousand, five hundred therefore you need to look at the tens to compare and order.

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Numbers to 100,000

Notes and Guidance

Children focus on numbers up to 100,000
They represent numbers on a place value grid, read and write numbers and place them on a number line to 100,000

Using a number line, they find numbers between two points, place a number and estimate where larger numbers will be.

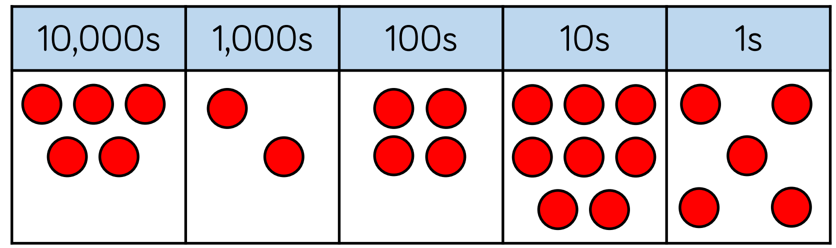
Mathematical Talk

How can the place value grid help you to add 10, 100 or 1,000 to any number?
How many digits change when you add 10, 100 or 1,000? Is it always the same number of digits that change?
How can we represent 65,048 on a number line?
How can we estimate a number on a number line if there are no divisions?
Do you need to count forwards and backwards to find out if a number is in a number sequence? Explain.

Varied Fluency

R

A number is shown in the place value grid.

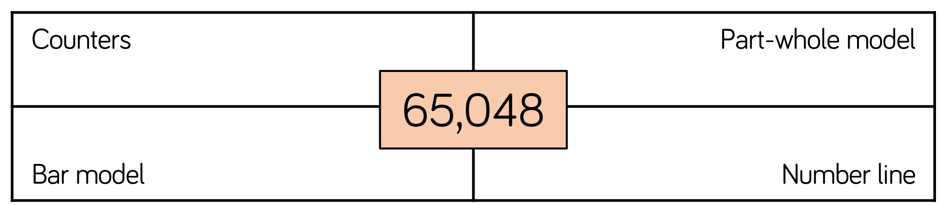


Write the number in figures and in words.

- Alex adds 10 to this number
- Tommy adds 100 to this number
- Eva adds 1,000 to this number

Write each of their new numbers in figures and in words.

Complete the grid to show the same number in different ways.



Complete the missing numbers.

59,000 = 50,000 + _____

_____ = 30,000 + 1,700 + 230

75,480 = _____ + 300 + _____

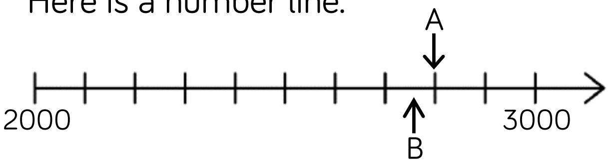
White Rose Maths

Numbers to 100,000

Reasoning and Problem Solving

R

Here is a number line.



What is the value of A?

B is 40 less than A.
What is the value of B?

C is 500 less than B.
Add C to the number line.

A = 2,800

B = 2,760

Here are three ways of partitioning 27,650

27 thousands and 650 ones
27 thousands, 5 hundreds and 150 ones
27 thousands and 65 tens

Write three more ways

Possible answers:

2 ten thousands, 6 hundreds and 5 tens

20 thousands, 7 thousands and 650 ones

Rosie counts forwards and backwards in 10s from 317

Circle the numbers Rosie will count.

427	997	−7
1,666	3,210	5,627
−23	7	−3

Explain why Rosie will not say the other numbers.

427
997
5,627
7
−3
−23

Any positive number will have to end in a 7

Any negative number will have to end in a 3

Numbers to One Million

Notes and Guidance

Children read, write and represent numbers to 1,000,000

They will recognise large numbers represented in a part-whole model, when they are partitioned in unfamiliar ways.

Children need to see numbers represented with counters on a place value grid, as well as drawing the counters.

Mathematical Talk

If one million is the whole, what could the parts be?

Show me 800,500 represented in three different ways.
Can 575,400 be partitioned into 4 parts in a different way?

Where do the commas go in the numbers?
How does the place value grid help you to represent large numbers?
Which columns will change in value when Eva adds 4 counters to the hundreds column?

Varied Fluency R

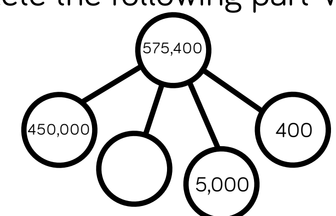
Thousands			Ones		
H	T	O	H	T	O

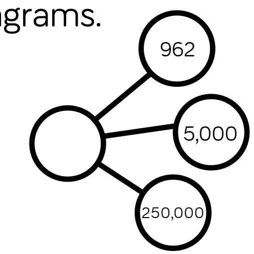
Use counters to make these numbers on the place value chart.

32,651 456,301 50,030

Can you say the numbers out loud?

Complete the following part-whole diagrams.





Eva has the following number.

Thousands			Ones		
H	T	O	H	T	O
	●●●●●●	●●●●●●●●●	●●●●●●	●●●●	●●

She adds 4 counters to the hundreds column.
What is her new number?

Numbers to One Million

Reasoning and Problem Solving

Describe the value of the digit 7 in each of the following numbers. How do you know?

407,338

700,491

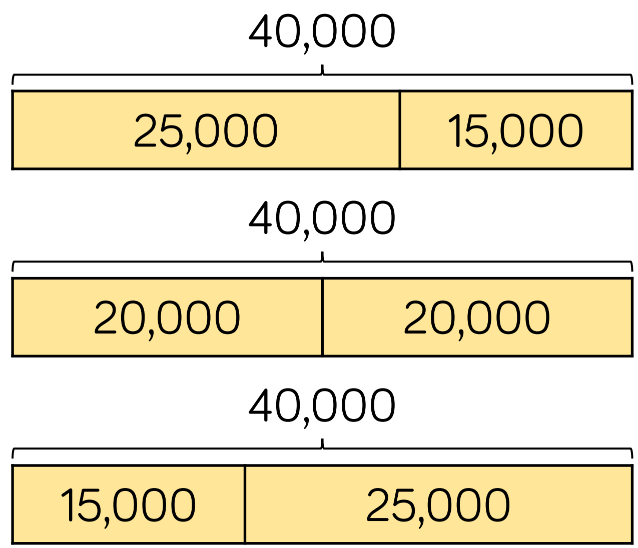
25,571

407,338: the value is 7 thousand. It is to the left of the hundreds column.

700,491: the value is 7 hundred thousand. It is a 6-digit number and there are 5 other numbers in place value columns to the right of this number.

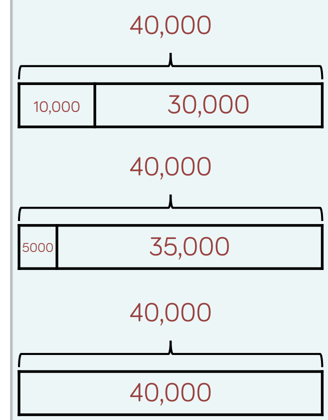
25,571: the value is 7 tens. It is one column to the left of the ones column.

The bar models are showing a pattern.



Draw the next three.

Create your own pattern of bar models for a partner to continue.



Numbers to Ten Million

Notes and Guidance

Children need to read, write and represent numbers to ten million in different ways.
Numbers do not always have to be in the millions – they should see a mixture of smaller and larger numbers, with up to seven digits. The repeating patterns of ones, tens, hundreds, ones of thousands, tens of thousands, hundreds of thousands could be discussed and linked to the placement of commas or other separators.

Mathematical Talk

Why is the zero in a number important when representing large numbers?

What strategies can you use to match the representation to the correct number?

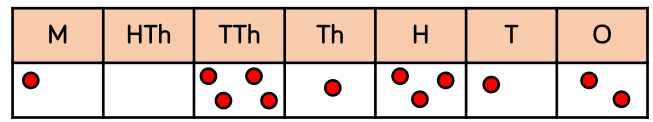
How many ways can you complete the partitioned number?

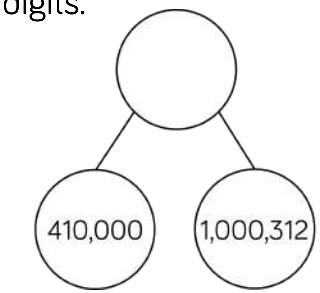
What strategy can you use to work out Teddy's new number?

Varied Fluency

- Match the representations to the numbers in digits.

One million, four hundred and one thousand, three hundred and twelve.





1,401,312 | 1,041,312 | 1,410,312

- Complete the missing numbers.

6,305,400 = ________ + 300,000 + ________ + 400

7,001,001 = 7,000,000 + ________ + ________

42,550 = ________ + ________ + ________ + 50

- Teddy's number is 306,042
He adds 5,000 to his number.
What is his new number?

Numbers to Ten Million

Reasoning and Problem Solving

Put a digit in the missing spaces to make the statement correct. 4,62 __ ,645 < 4,623,64 __ Is there more than one option? Can you find them all?	The first digit can be 0, 1, 2 or 3 When the first digit is 0, 1 or 2, the second digit can be any. When the first digit is 3, the second digit can be 6 or above.
Dora has the number 824,650 She subtracts forty thousand from her number. She thinks her new number is 820,650 Is she correct? Explain how you know.	Dora is incorrect because she has subtracted 4,000 not 40,000 Her answer should be 784,650

Use the digit cards and statements to work out my number. 0 3 3 5 5 6 7 • The ten thousands and hundreds have the same digit. • The hundred thousand digit is double the tens digit. • It is a six-digit number. • It is less than six hundred and fifty-five thousand. Is this the only possible solution?	Possible solutions: 653,530 653,537 650,537 650,533

Compare and Order

Notes and Guidance

Children will compare and order whole numbers up to ten million using numbers presented in different ways.

They should use the correct mathematical vocabulary (greater than/less than) alongside inequality symbols.

Mathematical Talk

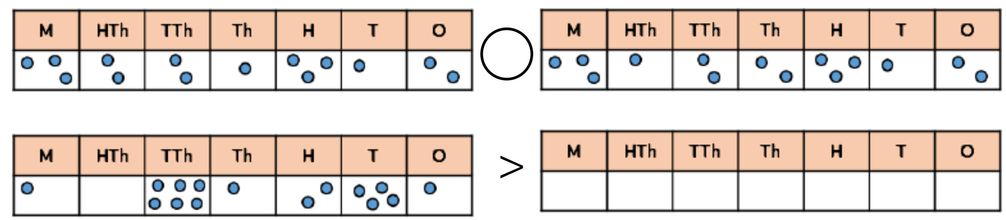
What is the value of each digit in the number?
What is the value of ______ in this number?

What is the value of the whole? Can you suggest other parts that make the whole?

What do you know about the covered number?
What could the number be? What must the number be? What can't the number be?

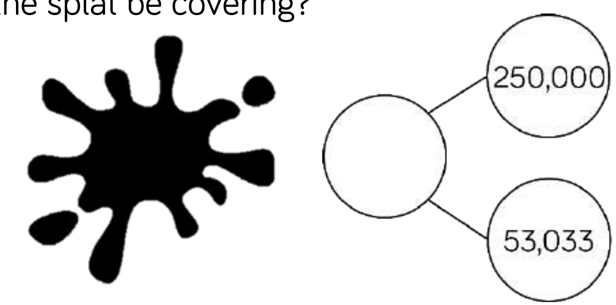
Varied Fluency

- Complete the statements to make them true.



- What number could the splat be covering?

Three hundred and thirteen thousand and thirty-three



Greatest → **Smallest**

- A house costs £250,000
  A motorised home costs £100,000
  A bungalow is priced halfway between the two.
  Work out the price of the bungalow.

Compare and Order

Reasoning and Problem Solving

Eva has ordered eight 6-digit numbers.

The smallest number is 345,900

The greatest number is 347,000

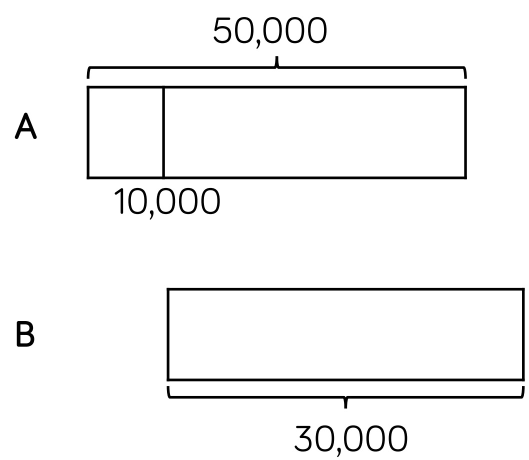
All the other numbers have a digit total of 20 and have no repeating digits.

What are the other six numbers?

Can you place all eight numbers in ascending order?

The other six numbers have to have a digit total of 20 and so must start with 346, _ _ _ because anything between 345,900 and 346,000 has a larger digit total. The final three digits have to add up to 7 so the solution is:
345,900
346,025
346,052
346,205
346,250
346,502
346,520
347,000

Jack draws bar model A.
His teacher asks him to draw another where the total is 30,000



Explain how you know bar B is inaccurate.

Bar B is inaccurate because it starts at 10,000 and finishes after 50,000 therefore it is longer than 40,000

Round to 10, 100 and 1,000

Notes and Guidance

Children build on their knowledge of rounding to 10, 100 and 1,000 from Year 4. They need to experience rounding up to and within 10,000

Children must understand that the column from the question and the column to the right of it are used e.g. when rounding 1,450 to the nearest hundred – look at the hundreds and tens columns. Number lines are a useful support.

Mathematical Talk

Which place value column do we need to look at when we round to the nearest 1,000?

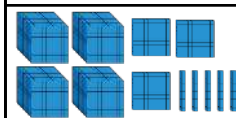
When is it best to round to the nearest 10? 100? 1,000?
Can you give an example of this?
Can you justify your reasoning?

Is there more than one solution?
Will the answers to the nearest 100 and 1,000 be the same or different for the different start numbers?

Varied Fluency

R

Complete the table.

Start Number	Rounded to the nearest 10	Rounded to the nearest 100	Rounded to the nearest 1,000
			
			
DCCLXIX			

For each number, find five numbers that round to it when rounding to the nearest 100

300 10,000 8,900

Complete the table.

Start Number	Nearest 10	Nearest 100	Nearest 1,000
365			
1,242			
	4,770		

Rounding to 10, 100 and 1,000

Reasoning and Problem Solving

R

My number rounded to the nearest 10 is 1,150
Rounded to the nearest 100 it is 1,200
Rounded to the nearest 1,000 it is 1,000

Jack

What could Jack's number be?

Can you find all of the possibilities?

1,150
1,151
1,152
1,153
1,154

2,567 to the nearest 100 is 2,500

Whitney

Do you agree with Whitney?
Explain why.

I do not agree with Whitney because 2,567 rounded to the nearest 100 is 2,600. I know this because if the tens digit is 5, 6, 7, 8 or 9 we round up to the next hundred.

Teddy

4,725 to the nearest 1,000 is 5,025

Explain the mistake Teddy has made.

Teddy has correctly changed four thousand to five thousand but has added the tens and the ones back on. When rounding to the nearest thousand, the answer is always a multiple of 1,000

Round within Ten Million

Notes and Guidance

Children build on their prior knowledge of rounding.

They will learn to round any number within ten million.

They use their knowledge of multiples and place value columns to work out which two numbers the number they are rounding sits between.

Mathematical Talk

Why do we round up when the following digit is 5 or above?
Which place value column do we need to look at when we round to the nearest 100,000?
What is the purpose of rounding?
When is it best to round to 1,000? 10,000?
Can you justify your reasoning?

What could/must/can't the missing digit be?
Explain how you know.

Varied Fluency

HTh	TTh	Th	H	T	O
●●●●●		●●●●●●●	●●●●●●●●●	●●●●●●●●	●●●●●●●●●

Round the number in the place value chart to:

- The nearest 10,000
- The nearest 100,000
- The nearest 1,000,000

Write five numbers that round to the following numbers when rounded to the nearest hundred thousand.

200,000   600,000   1,900,000

Complete the missing digits so that each number rounds to one hundred and thirty thousand when rounded to the nearest ten thousand.

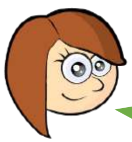
12 __ ,657   1 __ 1,999   13 __ ,001

Round within Ten Million

Reasoning and Problem Solving

My number is 1,350 when rounded to the nearest 10

Mo



My number is 1,400 when rounded to the nearest 100

Rosie

Both numbers are whole numbers.

What is the greatest possible difference between the two numbers?

The greatest possible difference is 104 (1,345 and 1,449)

Whitney rounded 2,215,678 to the nearest million and wrote 2,215,000

Can you explain to Whitney what mistake she has made?

There should be no non-zero digits in the columns after the millions column.

Miss Grogan gives out four number cards.

15,987	15,813	15,101	16,101

Four children each have a card and give a clue to what their number is.

Tommy says, “My number rounds to 16,000 to the nearest 1,000”

Alex says, “My number has one hundred.”

Jack says, “ My number is 15,990 when rounded to the nearest 10”

Dora says, “My number is 15,000 when rounded to the nearest 1,000”

Can you work out which child has which card?

Tommy: 15,813

Alex: 16,101

Jack: 15,987

Dora: 15,101

Negative Numbers

Notes and Guidance

Children continue their work on negative numbers from year 5 by counting forwards and backwards through zero.

They extend their learning by finding intervals across zero. Number lines, both vertical and horizontal are useful to support this, as these emphasise the position of zero.
Children need to see negative numbers in relevant contexts.

Mathematical Talk

Are all negative numbers whole numbers?
Why do the numbers on a number line mirror each other from 0?

Why does positive one add negative one equal zero?
Can you use a number line to show this?

Draw me a picture to show 5 subtract 8
Show 5 more than -2 on a number line.
Could Mo really afford the jumper? How do you know?

Varied Fluency

Use sandcastles $(+1)$ and holes (-1) to calculate.
Here is an example.

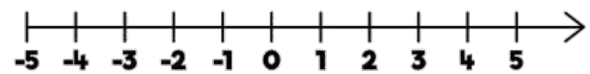
$-2 + 5 =$ 

Two sandcastles will fill two holes. There are three sandcastles left, therefore negative two add five is equal to three.

Use this method to solve:

$3 - 6$ $\quad$ $-7 + 8$ $\quad$ $5 - 9$

Use the number line to answer the questions.



- What is 6 less than 4?
- What is 5 more than -2?
- What is the difference between 3 and -3?

Mo has £17.50 in his bank account. He pays for a jumper which costs £30
How much does he have in his bank account now?

Negative Numbers

Reasoning and Problem Solving

A company decided to build offices over ground and underground. If we build from −20 to 20, we will have 40 floors.  Do you agree? Explain why.	No, there would be 41 floors because you need to count floor 0

When counting forwards in tens from any positive one-digit number, the last digit never changes. When counting backwards in tens from any positive one-digit number, the last digit does change. Can you find examples to show this? Explain why this happens.	Possible examples: 9, 19, 29, 39 etc. 9, −1, −11, −21 This happens because when you cross 0, the numbers mirror the positive side of the number line. Therefore, the final digit in the number changes and will make the number bond to 10

Autumn - Block 2

Four Operations

White Rose Maths

Overview

Small Steps

- Add whole numbers with more than 4 digits (R)
- Subtract whole numbers with more than 4 digits (R)
- Inverse operations (addition and subtraction) (R)
- Multi-step addition and subtraction problems (R)
- Add and subtract integers
- Multiply 4-digits by 1-digit (R)
- Multiply 2-digits (area model) (R)
- Multiply 2-digits by 2-digits (R)
- Multiply 3-digits by 2-digits (R)
- Multiply up to a 4-digit number by 2-digit number
- Divide 4-digits by 1-digit (R)
- Divide with remainders (R)
- Short division
- Division using factors

Notes for 2020/21

Year 6 assumes a lot of prior understanding of four operations. A deep understanding of these concepts are essential to help prepare children for secondary education and beyond.

Some children may not have had much practice in the last few months so we've included extended blocks and plenty of recap.

Overview

Small Steps

- Long division (1)
- Long division (2)
- Long division (3)
- Long division (4)
- Factors R
- Common factors
- Common multiples
- Primes to 100
- Squares and cubes
- Order of operations
- Mental calculations and estimation
- Reason from known facts

Notes for 2020/21

Year 6 assumes a lot of prior understanding of four operations. A deep understanding of these concepts are essential to help prepare children for secondary education and beyond.

Some children may not have had much practice in the last few months so we've included extended blocks and plenty of recap.

Add More than 4-digits

Notes and Guidance

Children will build upon previous learning of column addition. They will now look at numbers with more than four digits and use their place value knowledge to line the numbers up accurately.
Children use a range of manipulatives to demonstrate their understanding and use pictorial representations to support their problem solving.

Mathematical Talk

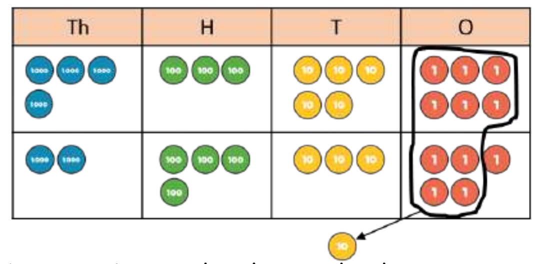
Will you have to exchange? How do you know which columns will be affected?

Does it matter that the two numbers don't have the same amount of digits?

Which number goes on top in the calculation? Does it affect the answer?

Varied Fluency

Ron uses place value counters to calculate 4,356 + 2,435



	Th	H	T	O
	4	3	5	6
+	2	4	3	5
	6	7	9	1
			1	

Use Ron's method to calculate:

	3	2	4	6	1
+		4	3	5	2

	4	8	2	7	6
+		5	6	1	3

Jack, Rosie and Eva are playing a computer game. Jack has 3,452 points, Rosie has 4,039 points and Eva has 10,989 points.

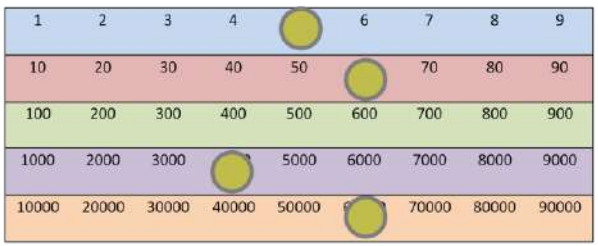
How many points do Jack and Rosie have altogether?
How many points do Rosie and Eva have altogether?
How many points do Jack and Eva have altogether?
How many points do Jack, Rosie and Eva have altogether?

Add More than 4-digits

Reasoning and Problem Solving

Amir is discovering numbers on a Gattegno chart.

He makes this number.



Amir moves one counter three spaces on a horizontal line to create a new number.

When he adds this to his original number he gets 131,130

Which counter did he move?

He moved the counter on the thousands row, he moved it from 4,000 to 7,000

Work out the missing numbers.

	?	4	?	3	?
+	2	?	5	?	2
	7	8	5	2	9

54,937 + 23,592 = 78,529

Subtract More than 4-digits

Notes and Guidance

Building on Year 4 experience, children use their knowledge of subtracting using the formal column method to subtract numbers with more than four digits. Children will be focusing on exchange and will be concentrating on the correct place value.
It is important that children know when an exchange is and isn't needed. Children need to experience '0' as a place holder.

Mathematical Talk

Why is it important that we start subtracting the smallest place value first?

Does it matter which number goes on top? Why? Will you have to exchange? How do you know which columns will be affected?

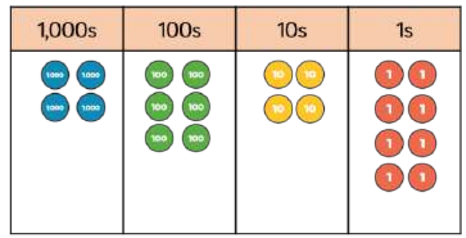
Does it matter that the two numbers don't have the same amount of digits?

Varied Fluency

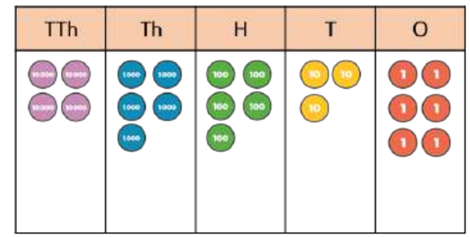
R

Calculate:

4,648 – 2,347



45,536 – 8,426



Represent each problem as a bar model, and solve them.

A plane is flying at 29,456 feet.
During the flight the plane descends 8,896 feet.
What height is the plane now flying at?

Tommy earns £37,506 pounds a year.
Dora earns £22,819 a year.
How much more money does Tommy earn than Dora?

There are 83,065 fans at a football match.
45,927 fans are male. How many fans are female?

Subtract More than 4-digits

Reasoning and Problem Solving

R

Eva makes a 5-digit number. Mo makes a 4-digit number. The difference between their numbers is 3,465 What could their numbers be?	Possible answers: 9,658 and 14,023 12,654 and 8,289 5,635 and 10,000 Etc.

Rosie completes this subtraction incorrectly. $$\begin{array}{r} 28701 \\ -\ 7621 \\ \hline 21180 \\ \hline \end{array}$$ Explain the mistake to Rosie and correct it for her.	Rosie did not write down the exchange she made when she exchanged 1 hundred for 10 tens. This means she still had 7 hundreds subtract 6 hundreds when she should have 6 hundreds subtract 6 hundreds. The correct answer is 21,080

Inverse Operations

Notes and Guidance

In this small step, children will use their knowledge of addition and subtraction to check their workings to ensure accuracy.

They use the commutative law to see that addition can be done in any order but subtraction cannot.

Mathematical Talk

How can you tell if your answer is sensible?

What is the inverse of addition?

What is the inverse of subtraction?

Varied Fluency

R

- When calculating 17,468 – 8,947, which answer gives the corresponding addition question?

  8,947 + 8,631 = 17,468

  8,947 + 8,521 = 17,468

  8,251 + 8,947 = 17,468

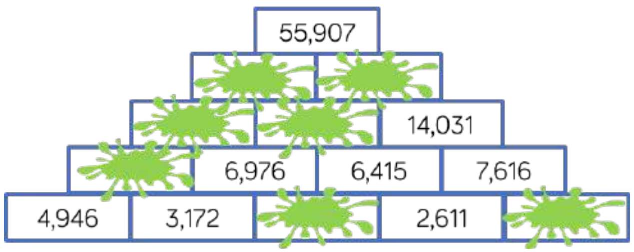
- I'm thinking of a number.
  After I add 5,241 and subtract 352, my number is 9,485
  What was my original number?

- Eva and Dexter are playing a computer game.
  Eva's high score is 8,524
  Dexter's high score is greater than Eva's.
  The total of both of their scores is 19,384
  What is Dexter's high score?

Inverse Operations

Reasoning and Problem Solving

Complete the pyramid using addition and subtraction.



From left to right:

Bottom row:
3,804, 5,005

Second row:
8,118

Third row:
15,094, 13,391

Fourth row:
28,485, 27,422

Mo, Whitney, Teddy and Eva collect marbles.

Mo: I have 1,648 marbles.

Whitney: I have double the amount of marbles Mo has.

Teddy: I have half the amount of marbles Mo has.

In total they have 8,524 marbles between them.
How many does Eva have?

Eva has 2,756 marbles.

Multi-step Problems

Notes and Guidance

In this small step children will be using their knowledge of addition and subtraction to solve multi-step problems.

The problems will appear in different contexts and in different forms i.e. bar models and word problems.

Mathematical Talk

What is the key vocabulary in the question?

What are the key bits of information?

Can we put this information into a model?

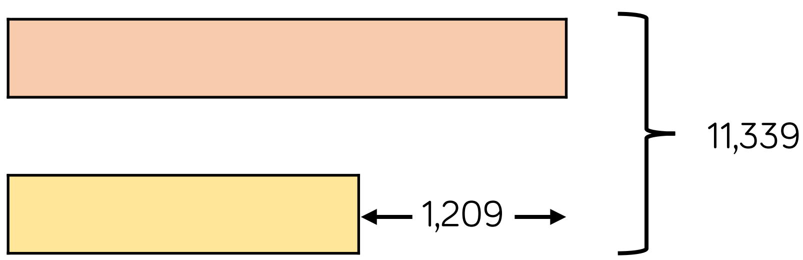
Which operations do we need to use?

Varied Fluency

- When Annie opened her book, she saw two numbered pages.
  The sum of these two pages was 317
  What would the next page number be?

- Adam is twice as old as Barry.
  Charlie is 3 years younger than Barry.
  The sum of all their ages is 53.
  How old is Barry?

- The sum of two numbers is 11,339
  The difference between the same two numbers is 1,209
  Use the bar model to help you find the numbers.



Multi-step Problems

Reasoning and Problem Solving

A milkman has 250 bottles of milk.

He collects another 160 from the dairy, and delivers 375 during the day.

How many does he have left?

My method:

375 – 250 = 125

125 + 160 = 285

Do you agree with Tommy?
Explain why.

Tommy is wrong. He should have added 250 and 160, then subtracted 375 from the answer.

There are 35 bottles of milk remaining.

On Monday, Whitney was paid £114

On Tuesday, she was paid £27 more than on Monday.

On Wednesday, she was paid £27 less than on Monday.

How much was Whitney paid in total?

How many calculations did you do?

Is there a more efficient method?

£342

Children might add 114 and 27, subtract 27 from 114 and then add their numbers.

A more efficient method is to recognise that the '£27 more' and '£27 less' cancel out so they can just multiply £114 by three.

Add & Subtract Integers

Notes and Guidance

Children consolidate their knowledge of column addition and subtraction, reinforcing the language of 'exchange' etc. After showing confidence with smaller numbers, children should progress to multi-digit calculations. Children will consider whether the column method is always appropriate e.g. when adding 999, it is easier to add 1,000 then subtract 1

They use these skills to solve multi-step problems in a range of contexts.

Mathematical Talk

What happens when there is more than 9 in a place value column?

Can you make an exchange between columns?

How can we find the missing digits? Can we use the inverse?

Is the column method always the best method?

When should we use mental methods?

Varied Fluency

- Calculate.

	3	4	6	2	1
+	2	5	7	3	4

67,832 + 5,258

	4	7	6	1	3	2	5
–		9	3	8	0	5	2

834,501 – 299,999

- A four bedroom house costs £450,000
A three bedroom house costs £201,000 less.
How much does the three bedroom house cost?
What method did you use to find the answer?

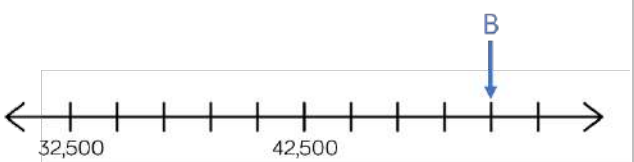
- Find the missing digits. What do you notice?

	5	2	2	4	7	?
+	3	?	5	9	0	4
	9	0	?	3	?	2

Add & Subtract Integers

Reasoning and Problem Solving

Find the difference between A and B.



A = 19,000

B = 50,500

The difference is 31,500

Here is a bar model.

A	B
631,255	

A is an odd number which rounds to 100,000 to the nearest ten thousand.
It has a digit total of 30

B is an even number which rounds to 500,000 to the nearest hundred thousand.
It has a digit total of 10

A and B are multiples of 5.

What are possible values of A and B?

Possible answer:

A = 99,255

B = 532,000

White Rose Maths

Multiply 4-digits by 1-digit

Notes and Guidance

Children build on previous steps to represent a 4-digit number multiplied by a 1-digit number using concrete manipulatives.
Teachers should be aware of misconceptions arising from using 0 as a place holder in the hundreds, tens or ones column.
Children then move on to explore multiplication with exchange in one, and then more than one column.

Mathematical Talk

Why is it important to set out multiplication using columns?

Explain the value of each digit in your calculation.

How do we show there is nothing in a place value column?

What do we do if there are ten or more counters in a place value column?

Which part of the multiplication is the product?

Varied Fluency

R

- Complete the calculation.

Thousands	Hundreds	Tens	Ones
1000		10 10	1 1 1
1000		10 10	1 1 1
1000		10 10	1 1 1

	Th	H	T	O
	1	0	2	3
×				3

- Write the multiplication calculation represented and find the answer.

Thousands	Hundreds	Tens	Ones
1000 1000	100		1 1 1 1 1 1
1000 1000	100		1 1 1 1 1 1

Remember if there are ten or more counters in a column, you need to make an exchange.

- Annie earns £1,325 per week.
How much would he earn in 4 weeks?

Thousands	Hundreds	Tens	Ones
1000	100 100 100	10 10	1 1 1 1 1
1000	100 100 100	10 10	1 1 1 1 1
1000	100 100 100	10 10	1 1 1 1 1
1000	100 100 100	10 10	1 1 1 1 1

	Th	H	T	O
	1	3	2	5
×				4

Multiply 4-digits by 1-digit

Reasoning and Problem Solving

R

Alex calculated 1,432 × 4

Here is her answer.

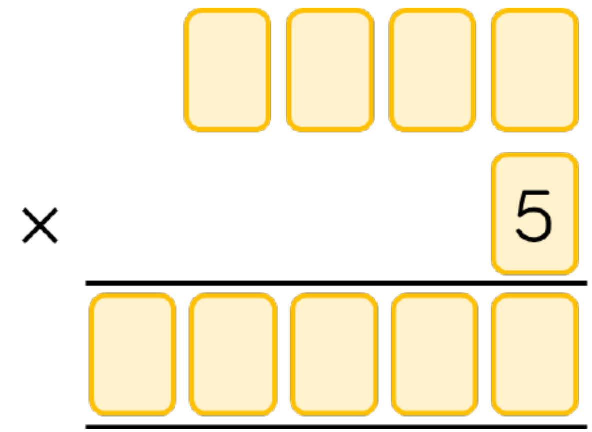
	Th	H	T	O
	1	4	3	2
×				4
	4	16	12	8

1,432 × 4 = 416,128

Can you explain what Alex has done wrong?

Alex has not exchanged when she has got 10 or more in the tens and hundreds columns.

Can you work out the missing numbers using the clues?



- The 4 digits being multiplied by 5 are consecutive numbers.
- The first 2 digits of the product are the same.
- The fourth and fifth digits of the answer add to make the third.

2,345 × 5 = 11,725

Multiply 2-digits (Area Model)

Notes and Guidance

Children use Base 10 to represent the area model of multiplication, which will enable them to see the size and scale linked to multiplying.

Children will then move on to representing multiplication more abstractly with place value counters and then numbers.

Mathematical Talk

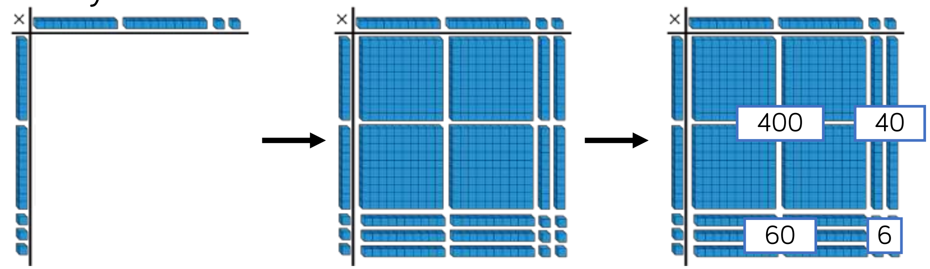
What are we multiplying?
How can we partition these numbers?

Where can we see 20 × 20?
What does the 40 represent?

What's the same and what's different between the three representations (Base 10, place value counters, grid)?

Varied Fluency

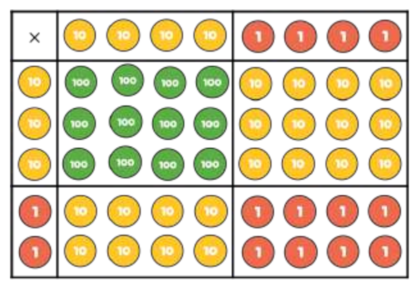
Whitney uses Base 10 to calculate 23 × 22



How could you adapt your Base 10 model to calculate these:

32 × 24   25 × 32   35 × 32

Rosie adapts the Base 10 method to calculate 44 × 32



×	40	4
30	1,200	120
2	80	8

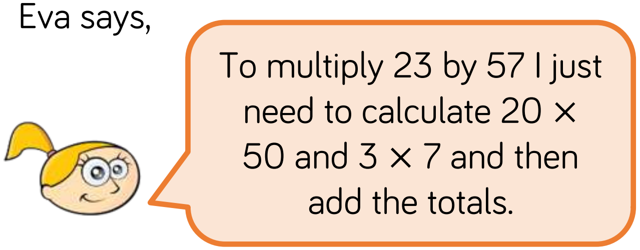
Compare using place value counters and a grid to calculate:

45 × 42   52 × 24   34 × 43

Multiply 2-digits (Area Model)

Reasoning and Problem Solving

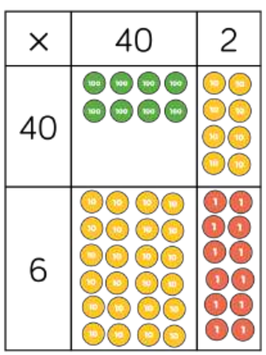
Eva says,



What mistake has Eva made?
Explain your answer.

Eva's calculation does not include 20 × 7 and 50 × 3
Children can show this with concrete or pictorial representations.

Amir hasn't finished his calculation.
Complete the missing information and record the calculation with an answer.



Amir needs 8 more hundreds, 40 × 40 = 1,600 and he only has 800

His calculation is 42 × 46 = 1,932

Farmer Ron has a field that measures 53 m long and 25 m wide.

Farmer Annie has a field that measures 52 m long and 26 m wide.

Dora thinks that they will have the same area because the numbers have only changed by one digit each.

Do you agree? Prove it.

Dora is wrong. Children may prove this with concrete or pictorial representations.

Multiply 2-digits by 2-digits

Notes and Guidance

Children will move on from the area model and work towards more formal multiplication methods.

They will start by exploring the role of the zero in the column method and understand its importance.

Children should understand what is happening within each step of the calculation process.

Mathematical Talk

Why is the zero important?

What numbers are being multiplied in the first line and in the second line?

When do we need to make an exchange?

What can we exchange if the product is 42 ones?

If we know what 38 × 12 is equal to, how else could we work out 39 × 12?

Varied Fluency

Complete the calculation to work out 23 × 14

		2	3	
×		1	4	
		9 [1]	2	(23 × 4)
	2	3	0	(23 × 10)

Use this method to calculate:

34 × 26   58 × 15   72 × 35

Complete to solve the calculation.

		4	6	
×		2	7	
	3	2 [4]	2	(__ × __)
	9 [1]	2	0	(__ × __)

Use this method to calculate:

27 × 39   46 × 55   94 × 49

Calculate:

38 × 12   39 × 12   38 × 11

What's the same? What's different?

Multiply 2-digits by 2-digits

Reasoning and Problem Solving

R

Tommy says,

It is not possible to make 999 by multiplying two 2-digit numbers.

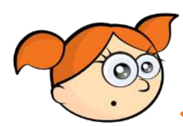
Do you agree?
Explain your answer.

Children may use a trial and error approach during which they'll further develop their multiplication skills.
They will find that Tommy is wrong because 27 × 37 is equal to 999

Amir has multiplied 47 by 36

		4	7
×		3	6
	2	8 $_{4}$	2
	1	4 $_{2}$	1
	3	2	3

Alex says,



Amir is wrong because the answer should be 1,692 not 323

Who is correct?
What mistake has been made?

Alex is correct.
Amir has forgotten to use zero as a place holder when multiplying by 3 tens.

Multiply 3-digits by 2-digits

Notes and Guidance

Children will extend their multiplication skills to multiplying 3-digit numbers by 2-digit numbers. They will use multiplication to find area and solve multi-step problems.
Methods previously explored are still useful e.g. using an area model.

Mathematical Talk

Why is the zero important?

What numbers are being multiplied in the first line and the second line?

When do we need to make an exchange?

What happens if there is an exchange in the last step of the calculation?

Varied Fluency

R

Complete:

		1	3	2	
×			1	4	
		5	2 [1]	8	(132 × 4)
	1	3	2	0	(132 × 10)

Use this method to calculate:

264 × 14 264 × 28

What do you notice about your answers?

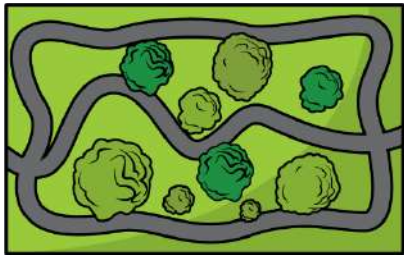
Calculate:

637 × 24

573 × 28

573 × 82

A playground is 128 yards by 73 yards.



Calculate the area of the playground.

Multiply 3-digits by 2-digits

Reasoning and Problem Solving

R

$22 \times 111 = 2442$

$23 \times 111 = 2553$

$24 \times 111 = 2664$

What do you think the answer to 25×111 will be?

What do you notice?

Does this always work?

The pattern stops at up to 28×111 because exchanges need to take place in the addition step.

Pencils come in boxes of 64
A school bought 270 boxes.
Rulers come in packs of 46
A school bought 720 packs.
How many more rulers were ordered than pencils?

15,840

Here are examples of Dexter's maths work.

			9	8	7
×				7	6
		5	${}_{5}9$	${}_{4}2$	2
		6	${}_{6}9$	${}_{4}0$	9
	1	${}_{1}2$	8	${}_{1}3$	1

			3	2	4
×				7	8
		${}_{2}$	${}_{1}5$	${}_{3}9$	2
	${}_{2}$	${}_{1}2$	${}_{2}6$	8	0
		3	2	7	2

He has made a mistake in each question.

Can you spot it and explain why it's wrong?

Correct each calculation.

In his first calculation, Dexter has forgotten to use a zero when multiplying by 7 tens.
It should have been
$987 \times 76 = 75,012$

In the second calculation, Dexter has not included his final exchanges.
$324 \times 8 = \underline{2},592$
$324 \times 70 = \underline{2}2,680$
The final answer should have been 25,272

Multiply 4-digits by 2-digits

Notes and Guidance

Children consolidate their knowledge of column multiplication, multiplying numbers with up to 4 digits by a 2-digit number. It may be useful to revise multiplication by a single digit first, and then 2- and 3- digit numbers before moving on when ready to the largest calculations.
They use these skills to solve multi-step problems in a range of contexts.

Mathematical Talk

What is important to remember as we begin multiplying by the tens number?

How would you draw the calculation?

Can the inverse operation be used?

Is there a different strategy that you could use?

Varied Fluency

- Calculate.

	4	2	6	7
×			3	4

	3	0	4	6
×			7	3

$5,734 \times 26$

- Jack made cookies for a bake sale.
He made 345 cookies.
The recipe says that he should have 17 raisins in each cookie.

How many raisins did he use altogether?

- Work out the missing number.

$6 \times 35 = \_\_\_\_ \times 5$

Multiply 4-digits by 2-digits

Reasoning and Problem Solving

True or False?

- 5,463 × 18 = 18 × 5,463

True

- I can find the answer to 1,100 × 28 by calculating 1,100 × 30 and subtracting 2 lots of 1,100

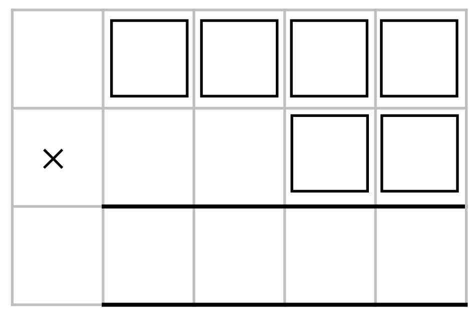
True

- 702 × 9 = 701 × 10

False

2 3 4 5 7 8

Place the digits in the boxes to make the largest product.



$$\begin{array}{r} 8432 \\ \times \quad 75 \\ \hline 632000 \\ \hline \end{array}$$

White Rose Maths

Divide 4-digits by 1-digit

Notes and Guidance

Children use their knowledge from Year 4 of dividing 3-digits numbers by a 1-digit number to divide up to 4-digit numbers by a 1-digit number.

They use place value counters to partition their number and then group to develop their understanding of the short division method.

Mathematical Talk

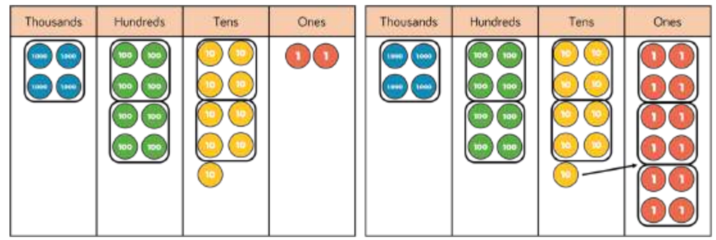
How many groups of 4 thousands are there in 4 thousands?
How many groups of 4 hundreds are there in 8 hundreds?
How many groups of 4 tens are there in 9 tens?
What can we do with the remaining ten?
How many groups of 4 ones are there in 12 ones?

Do I need to solve both calculations to compare the divisions?

Varied Fluency

R

- Here is a method to calculate 4,892 divided by 4 using place value counters and short division.



	1	2	2	3
4	4	8	9	^{1}2

Use this method to calculate:

6,610 ÷ 5   2,472 ÷ 3   9,360 ÷ 4

- Mr Porter has saved £8,934
  He shares it equally between his three grandchildren.
  How much do they each receive?

- Use <, > or = to make the statements correct.

3,495 ÷ 5	◯	3,495 ÷ 3
8,064 ÷ 7	◯	9,198 ÷ 7
7,428 ÷ 4	◯	5,685 ÷ 5

Divide 4-digits by 1-digit

Reasoning and Problem Solving

Jack is calculating 2,240 ÷ 7

He says you can't do it because 7 is larger than all of the digits in the number.

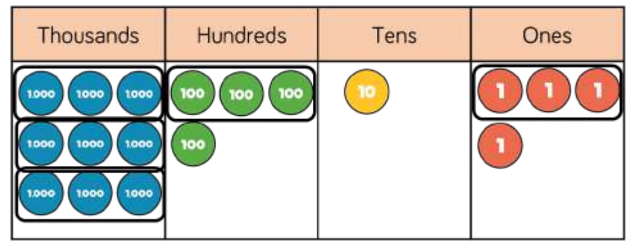
Do you agree with Jack?
Explain your answer.

Jack is incorrect.
You can exchange between columns.
You can't make a group of 7 thousands out of 2 thousand, but you can make groups of 7 hundreds out of 22 hundreds.

The answer is 320

Spot the Mistake

Explain and correct the working.



	3	1	0	1
3	9	4	1	4

There is no exchanging between columns within the calculation.
The final answer should have been 3,138

Divide with Remainders

Notes and Guidance

Children continue to use place value counters to partition and then group their number to further develop their understanding of the short division method.

They start to focus on remainders and build on their learning from Year 4 to understand remainders in context. They do not represent their remainder as a fraction at this point.

Mathematical Talk

If we can't make a group in this column, what do we do?

What happens if we can't group the ones equally?

In this number story, what does the remainder mean?

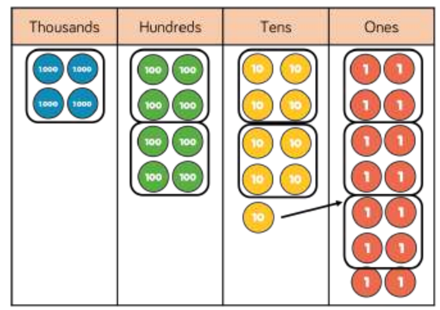
When would we round the remainder up or down?

In which context would we just focus on the remainder?

Varied Fluency

R

- Here is a method to solve 4,894 divided by 4 using place value counters and short division.



	1	2	2	3		
4	4	8	9	^{1}4		r2

Use this method to calculate:

6,613 ÷ 5   2,471 ÷ 3   9,363 ÷ 4

- Muffins are packed in trays of 6 in a factory.
  In one day, the factory makes 5,623 muffins.
  How many trays do they need?
  How many trays will be full?
  Why are your answers different?

- For the calculation 8,035 ÷ 4
  - Write a number story where you round the remainder up.
  - Write a number story where you round the remainder down.
  - Write a number story where you have to find the remainder.

Divide with Remainders

Reasoning and Problem Solving

I am thinking of a 3-digit number.

When it is divided by 9, the remainder is 3

When it is divided by 2, the remainder is 1

When it is divided by 5, the remainder is 4

What is my number?

Possible answers:

129	219
309	399
489	579
669	759
849	939

Encourage children to think about the properties of numbers that work for each individual statement.
This will help decide the best starting point.

Always, Sometimes, Never?

A three-digit number made of consecutive descending digits divided by the next descending digit always has a remainder of 1

$765 \div 4 = 191$ remainder 1

How many possible examples can you find?

Sometimes

Possible answers:

$432 \div 1 = 432$ r 0
$543 \div 2 = 271$ r 1
$654 \div 3 = 218$ r 0
$765 \div 4 = 191$ r 1
$876 \div 5 = 175$ r 1
$987 \div 6 = 164$ r 3

Short Division

Notes and Guidance

Children build on their understanding of dividing up to 4-digits by 1-digit by now dividing by up to 2-digits. They use the short division method and focus on the grouping structure of division. Teachers may encourage children to list multiples of the divisor (number that we are dividing by) to help them solve the division more easily. Children should experience contexts where the answer "4 r 1" means both 4 complete boxes or 5 boxes will be needed.

Mathematical Talk

In the hundreds column, how many groups of 5 are in 7? Are there are any hundreds remaining? What do we do next?

In the thousands column, there are no groups of three in 1 What do we do?

Why is the context of the question important when deciding how to round the remainders after a division?

Varied Fluency

- Calculate using short division.

5	7	2	5

3	1	9	3	8

1	2	6	0	3	6

3,612 ÷ 14

List the multiples of the divisors to help you calculate.

- A limousine company allows 14 people per limousine.

  How many limousines are needed for 230 people?

- Year 6 has 2,356 pencil crayons for the year.

  They put them in bundles, with 12 in each bundle.

  How many complete bundles can be made?

Short Division

Reasoning and Problem Solving

Find the missing digits.

0 4 1 [ ] r3

4 | 1 [ ] 5 9

0 4 1 4 r3

4 | 1 6 5 9

Here are two calculations.

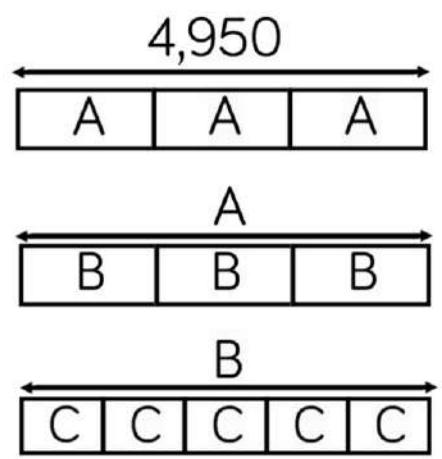
$A = 396 \div 11$

$B = 832 \div 13$

Find the difference between A and B.

$396 \div 11 = 36$
$832 \div 13 = 64$
$64 - 36 = 28$

Work out the value of C.
(The bar models are not drawn to scale)



$4{,}950 \div 3 = 1{,}650$

$1{,}650 \div 3 = 550$

$550 \div 5 = 110$

Division using Factors

Notes and Guidance

Children use their number sense, specifically their knowledge of factors, to be able to see relationships between the dividend (number being divided) and the divisor (number that the dividend is being divided by).

Beginning with multiples of 10 will allow children to see these relationships, before moving to other multiples.

Mathematical Talk

What is a factor?

How does using factor pairs help us to answer division questions?

Do you notice any patterns?

Does using factor pairs always work?

Is there more than one way to solve a calculation using factor pairs?

What methods can be used to check your working out?

Varied Fluency

- Calculate 780 ÷ 20

  Now calculate 780 ÷ 10 ÷ 2

  What do you notice? Why does this work?

  Use the same method to calculate 480 ÷ 60

- Use factors to help you calculate.

  4,320 ÷ 15

- Eggs are put into boxes.
  Each box holds 12 eggs.
  A farmer has 648 eggs that need to go in the boxes.

  How many boxes will he fill?

Division using Factors

Reasoning and Problem Solving

Calculate: • 1,248 ÷ 48 • 1,248 ÷ 24 • 1,248 ÷ 12 What did you do each time? What was your strategy? What do you notice? Why?	26 52 104 Children should recognise that when the dividend is halved, the answer (quotient) is doubled.
Tommy says,  To calculate 4,320 ÷ 15 I will first divide 4,320 by 5 then divide the answer by 10 Do you agree? Explain why.	Tommy is wrong: he has partitioned 15 when he should have used factor pairs. He could have used factor pairs 5 and 3 and divided by 5 then 3 (or 3 then 5).

Class 6 are calculating 7,848 ÷ 24 The children decide which factor pairs to use. Here are some of their suggestions: • 2 and 12 • 1 and 24 • 4 and 6 • 10 and 14 Which will not give them the correct answer? Why? Use the correct factor pairs to calculate the answer. Is the answer the same each time? Which factor pair would be the least efficient to use? Why?	10 and 14 is incorrect because they are not factors of 24 (to get 10 and 14, 24 has been partitioned). The correct answer is 327 Children should get the same answer using all 3 factor pairs methods. Using the factor pair of 1 and 24 is the least efficient.

Long Division (1)

Notes and Guidance

Children are introduced to long division as a different method of dividing by a 2-digit number.

They divide 3-digit numbers by a 2-digit number without remainders, starting with a more expanded method (with multiples shown), before progressing to the more formal long division method.

Mathematical Talk

How can we use multiples to help us divide by a 2-digit number?

Why are we subtracting the totals from the dividend (starting number)? This question supports children to see division as repeated subtraction.

In long division, what does the arrow represent? (The movement of the next digit coming down to be divided).

Varied Fluency

		0	3	6	
1	2	4	3	2	
	–	3	6	0	(×30)
			7	2	
	–		7	2	(×6)
				0	

<u>Multiples of 12:</u>

12 × 1 = 12
12 × 2 = 24
12 × 3 = 36
12 × 4 = 48
12 × 5 = 60
12 × 6 = 72
12 × 7 = 84
12 × 8 = 96
12 × 7 = 108
12 × 10 = 120

Use this method to calculate:

765 ÷ 17   450 ÷ 15   702 ÷ 18

		0	3	6
1	2	4	3	2
	–	3	6	↓
			7	2
	–		7	2
				0

Use the long division method to calculate:

836 ÷ 11
798 ÷ 14
608 ÷ 19

Long Division (1)

Reasoning and Problem Solving

Odd One Out

Which is the odd one out?
Explain your answer.

$512 \div 16$

$672 \div 21$

$792 \div 24$

$792 \div 24 = 33$ so this is the odd one out as the other two give an answer of 32

Spot the Mistake

$855 \div 15 =$

		0	5	1	0	
1	5	8	5	5		
	–	7	5		(×	4)
		1	0	5		
	–	1	0	5	(×	10)
				0		

The mistake is that $105 \div 15$ is not equal to 10

$105 \div 15 = 7$ so the answer to the calculation is 57

Long Division (2)

Notes and Guidance

Building on using long division with 3-digit numbers, children divide 4-digit numbers by 2-digits using the long division method.

They use their knowledge of multiples and multiplying and dividing by 10 and 100 to calculate more efficiently.

Mathematical Talk

How can we use multiples to help us divide by a 2-digit number?

Why are we subtracting the totals from the dividend (starting number)? This question supports children to see division as repeated subtraction.

In long division, what does the arrow represent? (The movement of the next digit coming down to be divided).

Varied Fluency

Here is a division method.

	0	4	8	9	
15	7	3	3	5	
–	6	0	0	0	(×400)
	1	3	3	5	
–	1	2	0	0	(×80)
		1	3	5	
–		1	3	5	(×9)
				0	

Use this method to calculate:

2,208 ÷ 16   1,755 ÷ 45   1,536 ÷ 16

There are 1,989 footballers in a tournament.
Each team has 11 players and 2 substitutes.
How many teams are there in the tournament?

Long Division (2)

Reasoning and Problem Solving

Which calculation is harder?

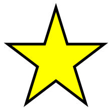
1,950 ÷ 13

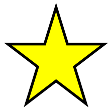
1,950 ÷ 15

Explain why.

Dividing by 13 is harder because 13 is prime so we cannot use factor knowledge to factorise it into smaller parts. The 13 times table is harder than the 15 times table because the 15 times table is related to the 5 times table whereas the 13 times table is not related to a more common times table (because 13 is prime).

6,120 ÷ 17 = 360

Explain how to use this fact to find 

6,480 ÷  = 360

6,480 is 360 more than 6,120, so there is 1 group of 360 more.

Therefore, there are 18 groups of 360, so the answer is 18

Long Division (3)

Notes and Guidance

Children now divide using long division where answers have remainders. After dividing, they check that the remainder is smaller than the divisor.

Children start to understand how to interpret the remainder e.g. **380 ÷ 12 = 31 r 8 could mean 31 full packs, or 32 packs needed depending on context.**

Mathematical Talk

How can we use multiples to help us divide?

What happens if we cannot divide the ones exactly by the divisor? How do we show what is left over?

Why are we subtracting the totals from the dividend (starting number)?

Why is the context of the question important when deciding how to round the remainders after a division?

Varied Fluency

Tommy uses this method to calculate 372 divided by 15
He has used his knowledge of multiples to help.

			2	4	r	1	2
1	5	3	7	2			
	–	3	0	0			
			7	2			
	–		6	0			
			1	2			

$1 \times 15 = 15$

$2 \times 15 = 30$

$3 \times 15 = 45$

$4 \times 15 = 60$

$5 \times 15 = 75$

$10 \times 15 = 150$

Use this method to calculate:

271 ÷ 17 **623 ÷ 21** **842 ÷ 32**

A school needs to buy 380 biscuits for parents' evening.
Biscuits are sold in packs of 12

How many packets will the school need to buy?

Long Division (3)

Reasoning and Problem Solving

Here are two calculation cards.

A = 396 ÷ 11

B = 832 ÷ 11

Whitney thinks there won't be a remainder for either calculation because 396 and 832 are both multiples of 11

Rosie disagrees, she has done the written calculations and says **one** of them has a remainder.

Who is correct? Explain your answer.

Rosie is correct because 832 is not a multiple of 11

396 ÷ 11 = 36

832 ÷ 11 = 75 r 7

576 children and 32 adults need transport for a school trip.
A coach holds 55 people.

Dora: "We need 10 coaches."

Eva: "We need 11 coaches."

Alex: "We need 12 coaches."

Who is correct? Explain how you know.

How many spare seats will there be?

Alex is correct.

There are 608 people altogether, 608 ÷ 55 = 11 r 3, so 12 coaches are needed.

On 12 coaches there will be 660 seats, because 55 × 12 = 660
660 − 608 = 52 spare seats.

Long Division (4)

Notes and Guidance

Children now divide four-digit numbers using long division where their answers have remainders. After dividing, they check that their remainder is smaller than their divisor.

Children start to understand when rounding is appropriate to use for interpreting the remainder and when the context means that it is not applicable.

Mathematical Talk

How can we use multiples to help us divide?

What happens if we cannot divide the ones exactly by the divisor? How do we show what is left over?

Why are we subtracting the totals from the dividend (starting number)? This question supports children to see division as repeated subtraction.

Does the remainder need to be rounded up or down?

Varied Fluency

Amir used this method to calculate 1,426 divided by 13

			1	0	9	r	9	
1	3	1	4	2	6			
	–	1	3	0	0			(× 100)
			1	2	6			
	–		1	1	7			(×9)
					9			

Use this method to calculate:

2,637 ÷ 16   4,453 ÷ 22   4,203 ÷ 18

A large bakery produces 7,849 biscuits in a day which are packed in boxes.
Each box holds 64 biscuits.

How many boxes are needed so all the biscuits are in a box?

Long Division (4)

Reasoning and Problem Solving

Class 6 are calculating three thousand, six hundred and thirty-three divided by twelve. Rosie says that she knows there will be a remainder without calculating. Is she correct? Explain your answer. What is the remainder?	Rosie is correct because 3,633 is odd and 12 is even, and all multiples of 12 are even because 12 is even. 3,633 ÷ 12 = 302 r 9, so the remainder is 9

Which numbers up to 20 can 4,236 be divided by without having a remainder? What do you notice about all the numbers?	1, 2, 3, 4, 6, 12 They are all factors of 12

Factors

Notes and Guidance

Children understand the relationship between multiplication and division and use arrays to show the relationship between them. Children learn that factors of a number multiply together to give that number, meaning that factors come in pairs. Factors are the whole numbers that you multiply together to get another whole number (factor × factor = product).

Mathematical Talk

How can you work in a systematic way to prove you have found all the factors?

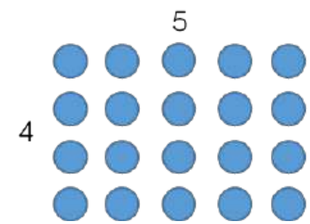
Do factors always come in pairs?

How can we use our multiplication and division facts to find factors?

Varied Fluency

R

- If you have twenty counters, how many different ways of arranging them can you find?



How many factors of twenty have you found by arranging your counters in different arrays?

- Circle the factors of 60

9, 6, 8, 4, 12, 5, 60, 15, 45

Which factors of 60 are not shown?

- Fill in the missing factors of 24

1 × ____ ____ × 12

3 × ____ ____ × ____

What do you notice about the order of the factors?
Use this method to find the factors of 42

Factors

Reasoning and Problem Solving

Here is Annie's method for finding factor pairs of 36

1	36
2	18
3	12
4	9
5	X
6	6

When do you put a cross next to a number?

How many factors does 36 have?

Use Annie's method to find all the factors of 64

If it is not a factor, put a cross.

36 has 9 factors.

Factors of 64:

1	64
2	32
3	X
4	16
5	X
6	X
7	X
8	8

Always, Sometimes, Never

- An even number has an even amount of factors.
- An odd number has an odd amount of factors.

Sometimes, e.g. 6 has four factors but 36 has nine.

Sometimes, e.g. 21 has four factors but 25 has three.

True or False?

The bigger the number, the more factors it has.

False. For example, 12 has 6 factors but 13 only has 2

Common Factors

Notes and Guidance

Children find the common factors of two numbers.

Some children may still need to use arrays and other representations at this stage but mental methods and knowledge of multiples should be encouraged.

They can show their results using Venn diagrams and tables.

Mathematical Talk

How do you know you have found all the factors of a given number?

Have you used a systematic approach?

Can you explain your system to a partner?

How does a Venn diagram show common factors?

Where are the common factors?

Varied Fluency

- Find the common factors of each pair of numbers.

  24 and 36

  20 and 30

  28 and 45

- Which number's factors make it the odd one out?

  12, 30, 54, 42, 32, 48

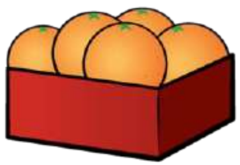
  Can you explain why?

- Two numbers have common factors of 4 and 9
  What could the numbers be?

Common Factors

Reasoning and Problem Solving

There are 49 pears and 56 oranges.

They need to be put into baskets of pears and baskets of oranges with an equal number of fruit in each basket.

Amir says,

There will be 8 pieces of fruit in each basket.

Jack says,

There will be 7 pieces of fruit in each basket.

Who is correct? Explain how you know.

Jack is correct. There will be seven pieces of fruit in each basket because 7 is a common factor of 49 and 56

Tommy has two pieces of string.

One is 160 cm long and the other is 200 cm long.

He cuts them into pieces of equal length.

What are the possible lengths the pieces of string could be?

The possible lengths are: 2, 4, 5, 8, 10, 20 and 40 cm.

Dora has 32 football cards that she is giving away to his friends.

She shares them equally between her friends.

How many friends could Dora have?

Dora could have 1, 2, 4, 8, 16 or 32 friends.

Common Multiples

Notes and Guidance

Building on knowledge of multiples, children find common multiples of numbers. They should continue to use visual representations to support their thinking.

They also use abstract methods to calculate multiples, including using numbers outside of those known in times table facts.

Mathematical Talk

Is the lowest common multiple of a pair of numbers always the product of them?

Can you think of any strategies to work out the lowest common multiples of different numbers?

When do numbers have common multiples that are lower than their product?

Varied Fluency

- On a 100 square, shade the first 5 multiples of 7 and then the first 8 multiples of 5

  What common multiple of 7 and 5 do you find?

  Use this number to find other common multiples of 7 and 5

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

- List 5 common multiples of 4 and 3

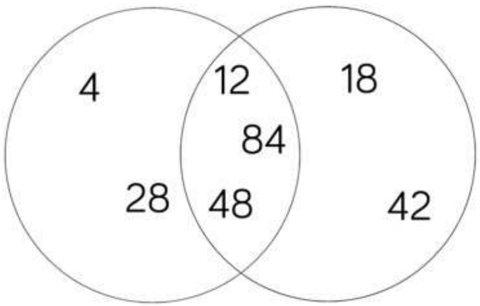
- Alex and Eva play football at the same local football pitches. Alex plays every 4 days and Eva plays every 6 days.

  They both played football today.

  After a fortnight, how many times will they have played football on the same day?

Common Multiples

Reasoning and Problem Solving

Work out the headings for the Venn diagram.  Add in one more number to each section. Can you find a square number that will go in the middle section of the Venn diagram?	Multiples of 4 Multiples of 6 144 is a square number that can go in the middle.
Annie is double her sister's age. They are both older than 20 but younger than 50 Their ages are both multiples of 7 What are their ages?	Annie is 42 and her sister is 21
A train starts running from Leeds to York at 7am. The last train leaves at midnight. Platform 1 has a train leaving from it every 12 minutes. Platform 2 has one leaving from it every 5 minutes. How many times in the day would there be a train leaving from both platforms at the same time?	18 times

Primes to 100

Notes and Guidance

Building on their learning in year 5, children should know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers.

They should be able to use their understanding of prime numbers to work out whether or not numbers up to 100 are prime. Using primes, they break a number down into its prime factors.

Mathematical Talk

What is a prime number?

What is a composite number?

How many factors does a prime number have?

Are all prime numbers odd?

Why is 1 not a prime number?

Why is 2 a prime number?

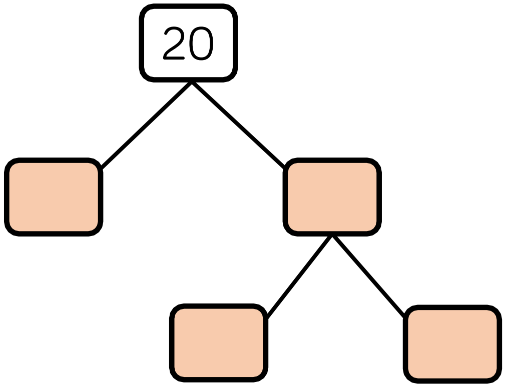
Varied Fluency

- List all of the prime numbers between 10 and 30

- The sum of two prime numbers is 36

  What are the numbers?

- All numbers can be broken down into prime factors.
  A prime factor tree can help us find them.
  Complete the prime factor tree for 20



Primes to 100

Reasoning and Problem Solving

Use the clues to work out the number. • It is greater than 10 • It is an odd number • It is not a prime number • It is less than 25 • It is a factor of 60	15
Shade in the multiples of 6 on a 100 square. What do you notice about the numbers either side of every multiple of 6? Eva says, "I noticed there is always a prime number next to a multiple of 6" Is she correct?	Both numbers are always odd. Yes, Eva is correct because at least one of the numbers either side of a multiple of 6 is always prime for numbers up to 100

Square & Cube Numbers

Notes and Guidance

Children have identified square and cube numbers previously and now explore the relationship between them, and solve problems involving them.
They need to experience sorting the numbers into different diagrams and look for patterns and relationships. They explore general statements regarding square and cube numbers. This step is a good opportunity to practise efficient mental methods of calculation.

Mathematical Talk

What do you notice about the sequence of square numbers?

What do you notice about the sequence of cube numbers?

Explore the pattern of the difference between the numbers.

Varied Fluency

- Use <, > or = to make the statements correct.

  3 cubed ◯ 4 squared

  8 squared ◯ 4 cubed

  11 squared ◯ 5 cubed

- This table shows square and cube numbers. Complete the table. Explain the relationships you can see between the numbers.

		1			1
					8
	3×3		3^3		27
	4×4			$4 \times 4 \times 4$	
		25	5^3		
				$6 \times 6 \times 6$	
8^2					

- ___ + 35 = 99

  210 − ___ = 41

  Which square numbers are missing from the calculations?

Square & Cube Numbers

Reasoning and Problem Solving

Place 5 odd and 5 even numbers in the table.

	Not Cubed	Cubed
Over 100		
100 or less		

Possible cube numbers to use:

1, 8, 27, 64, 125, 216, 343, 512, 729, 1,000

Jack says,

The smallest number that is both a square number and a cube number is 64

Do you agree with Jack? Explain why you agree or disagree.

Jack is incorrect. 1 is the smallest number that is both a square number ($1^2 = 1$) and cube number ($1^3 = 1$).

Shade in all the square numbers on a 100 square.

Now shade in multiples of 4

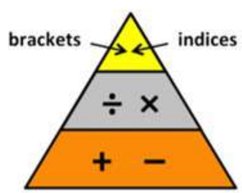
What do you notice?

Square numbers are always either a multiple of 4 or 1 more than a multiple of 4

Order of Operations

Notes and Guidance

Children will look at different operations within a calculation and consider how the order of operations affects the answer. Children will learn that, in mixed operation calculations, calculations are not carried out from left to right.
Children learn the convention that when there is no operation sign written this means multiply e.g. $4(2 + 1)$ means $4 \times (2 + 1)$. This image is useful when teaching the order of operations.



Mathematical Talk

Does it make a difference if you change the order in a mixed operation calculation?

What would happen if we did not use the brackets?

Would the answer be correct?

Why?

Varied Fluency

- Alex has 7 bags with 5 sweets in each bag.
  She adds one more sweet to each bag.
  Which calculation will work out how many sweets she now has in total? Explain your answer.

$$7 \times (5 + 1)$$

$$7 \times 5 + 1$$

- Teddy has completed this calculation and got an answer of 5

$$14 - 4 \times 2 \div 4 = 5$$

  Explain and correct his error.

- Add brackets and missing numbers to make the calculations correct.

$$6 + \_\_\_ \times 5 = 30$$

$$25 - 6 \times \_\_\_ = 38$$

Order of Operations

Reasoning and Problem Solving

Countdown Big numbers: 25, 50, 75, 100 Small numbers: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 Children randomly select 6 numbers. Reveal a target number. Children aim to make the target number ensuring they can write it as a single calculation using order of operations.	

Write different number sentences using the digits 3, 4, 5 and 8 before the equals sign that use: • One operation • Two operations with no brackets • Two operations with brackets	Possible solutions: $58 - 34 = 24$ $58 + 3 \times 4 = 60$ $5(8 - 3) + 4 = 29$

Mental Calculations

Notes and Guidance

We have included this small step separately to ensure that teachers emphasise this important skill. Discussions with children around efficient mental calculations and sensible estimations need to run through all steps.

Sometimes children are too quick to move to computational methods, when more efficient mental strategies should be used.

Mathematical Talk

Is there an easy and quick way to do this?

Can you use known facts to answer the problem?

Can you use rounding?

Does the solution need an exact answer?

How does knowing the approximate answer help with the calculation?

Varied Fluency

How could you change the order of these calculations to be able to perform them mentally?

$50 \times 16 \times 2$

$30 \times 12 \times 2$

$4 \times 17 \times 25$

Mo wants to buy a t-shirt for £9.99, socks for £1.49 and a belt for £8.99
He has £22 in his wallet.
How could he quickly check if he has enough money?

What number do you estimate is shown by arrow B when:

- A = 0 and C = 1,000
- A = 30 and C = 150
- A = −7 and C = 17
- A = 1 and C = 2
- A = 1,000 and C = 100,000

Mental Calculations

Reasoning and Problem Solving

Class 6 are calculating the total of 3,912 and 3,888

Alex says,

We can just double 3,900

Is Alex correct? Explain.

Alex is correct because 3,912 is 12 more than 3,900 and 3,888 is 12 less than 3,900

3,900 × 2 = 7,800

2,000 – 1,287

Here are three different strategies for this subtraction calculation:

Dora

I used the column method.

Tommy

I used my number bonds from 87 to 100 then from 1,300 to 2,000

Jack

I subtracted one from each number and then used the column method.

Whose method is most efficient?

Children share their ideas. Discuss how Dora's method is inefficient for this calculation because of the need to make multiple exchanges.

Jack's method is known as the 'constant difference' method and avoids exchanging.

Reason from Known Facts

Notes and Guidance

Children should use known facts from one calculation to determine the answer of another similar calculation without starting afresh.

They should use reasoning and apply their understanding of commutativity and inverse operations.

Mathematical Talk

What is the inverse?

When do you use the inverse?

How can we use multiplication/division facts to help us answer similar questions?

Varied Fluency

- Complete.

  $70 \div \_\_\_ = 7$　　$3.5 \times 10 = \_\_\_$

  $70 \div \_\_\_ = 3.5$　　$\_\_\_ = 3.5 \times 20$

  $70 \div \_\_\_ = 14$　　$\_\_\_ = 3.5 \times 2$

  Make a similar set of calculations using $90 \div 2 = 45$

- $5{,}138 \div 14 = 367$

  Use this to calculate 15×367

- $14 \times 8 = 112$

  Use this to calculate:

  - 1.4×8
  - 9×14

Reason from Known Facts

Reasoning and Problem Solving

$$3,565 + 2,250 = 5,815$$

Use this calculation to decide if the following calculations are true or false.

True or False?

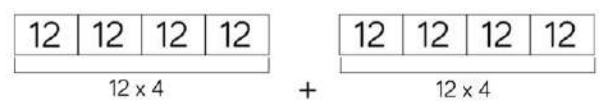
$4,565 + 1,250 = 5,815$	True
$5,815 - 2,250 = 3,565$	True
$4,815 - 2,565 = 2,250$	True
$3,595 + 2,220 = 5,845$	False

Which calculations will give an answer that is the same as the product of 12 and 8?

$3 \times 4 \times 8$

$12 \times 4 \times 2$

$2 \times 10 \times 8$



The product of 12 and 8 is 96

The 1st and 2nd calculations give an answer of 96 In the 1st calculation 12 has been factorised into 3 and 4, and in the 2nd calculation 8 has been factorised into 4 and 2

The third calculation gives an answer of 160

Autumn - Block 3

Fractions

Overview

Small Steps

- Equivalent fractions (R)
- Simplify fractions
- Improper fractions to mixed numbers (R)
- Mixed numbers to improper fractions (R)
- Fractions on a number line
- Compare and order (denominator)
- Compare and order (numerator)
- Add and subtract fractions (1)
- Add and subtract fractions (2)
- Add mixed numbers (R)
- Add fractions
- Subtract mixed numbers (R)
- Subtract fractions

Notes for 2020/21

Many children may have missed the block of learning from Y5 on fractions therefore we are suggesting recapping this.

Spend time ensuring children can add and subtract proper fractions, before moving onto mixed numbers.

These skills require understanding of equivalent fractions.

Overview

Small Steps

- Mixed addition and subtraction
- Multiply fractions by integers
- Multiply fractions by fractions
- Divide fractions by integers (1)
- Divide fractions by integers (2)
- Four rules with fractions
- Fraction of an amount
- Fraction of an amount – find the whole

Notes for 2020/21

Many children may have missed the block of learning from Y5 on fractions therefore we are suggesting recapping this.

Spend time ensuring children can add and subtract proper fractions, before moving onto mixed numbers.

These skills require understanding of equivalent fractions.

Equivalent Fractions

Notes and Guidance

Children explore equivalent fractions using models and concrete representations.

They use models to make the link to multiplication and division. Children then apply the abstract method to find equivalent fractions.

It is important children have the conceptual understanding before moving on to just using an abstract method.

Mathematical Talk

What equivalent fractions can we find by folding the paper? How can we record these?

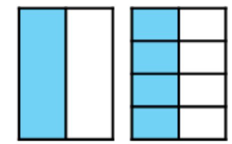
What is the same and what is different about the numerators and denominators in the equivalent fractions?

How does multiplication and division help us find equivalent fractions? Where can we see this in our model?

Varied Fluency

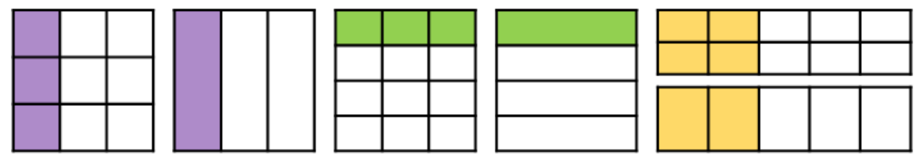
R

- Take two pieces of paper the same size.
Fold one piece into two equal pieces.
Fold the other into eight equal pieces.
What equivalent fractions can you find?

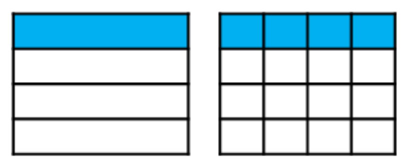


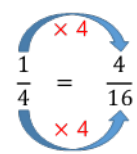
$\frac{1}{2} = \frac{4}{8}$

Use the models to write equivalent fractions.



- Eva uses the models and her multiplication and division skills to find equivalent fractions.





Use this method to find equivalent fractions to $\frac{2}{4}$, $\frac{3}{4}$ and $\frac{4}{4}$ where the denominator is 16

Eva uses the same approach to find equivalent fractions for these fractions. How will her method change?

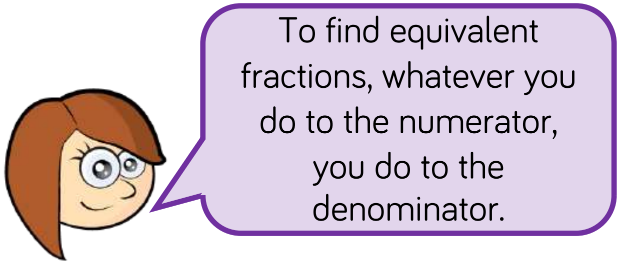
$\frac{4}{12} = \frac{\square}{3}$ $\frac{6}{12} = \frac{\square}{4}$ $\frac{6}{12} = \frac{\square}{2}$

Equivalent Fractions

Reasoning and Problem Solving

R

Rosie says,



Using her method, here are the equivalent fractions Rosie has found for $\frac{4}{8}$

$\frac{4}{8} = \frac{8}{16}$ $\frac{4}{8} = \frac{6}{10}$

$\frac{4}{8} = \frac{2}{4}$ $\frac{4}{8} = \frac{1}{5}$

Are all Rosie's fractions equivalent?
Does Rosie's method work?
Explain your reasons.

$\frac{4}{8} = \frac{1}{5}$ and $\frac{4}{8} = \frac{6}{10}$ are incorrect.

Rosie's method doesn't always work. It works when multiplying or dividing both the numerator or denominator but not when adding or subtracting the same thing to both.

Ron thinks you can only simplify even numbered fractions because you keep on halving the numerator and denominator until you get an odd number.

Do you agree?
Explain your answer.

Ron is wrong. For example $\frac{3}{9}$ can be simplified to $\frac{1}{3}$ and these are all odd numbers.

Here are some fraction cards.
All of the fractions are equivalent.

$\frac{4}{A}$ $\frac{B}{C}$ $\frac{20}{50}$

A + B = 16
Calculate the value of C.

A = 10
B = 6
C = 15

Simplify Fractions

Notes and Guidance

Children use their understanding of the highest common factor to simplify fractions, building on their knowledge of equivalent fractions in earlier years.
Children apply their understanding when calculating with fractions and simplifying their answers. Encourage children to use pictorial representations to support simplifying e.g. a fraction wall.

Mathematical Talk

Can you make a list of the factors for each number?
Which numbers appear in both lists? What do we call these (common factors)?
What is the highest common factor of the numerator and denominator?
Is a simplified fraction always equivalent to the original fraction? Why?
If the HCF of the numerator and denominator is 1, can it be simplified?

Varied Fluency

- Alex is simplifying $\frac{8}{12}$ by dividing the numerator and denominator by their highest common factor.

Factors of 8: 1 , 2 , **4** , 8
Factors of 12: 1 , 2 , 3 , **4** , 6 , 12
4 is the highest common factor.

÷ 4

$$\frac{8}{12} = \frac{2}{3}$$

÷ 4

Use Alex's method to simplify these fractions:

$\frac{6}{9}$ $\frac{6}{18}$ $\frac{10}{18}$ $\frac{10}{15}$ $\frac{15}{50}$

- Mo has 3 boxes of chocolates. 2 boxes are full and one box is $\frac{4}{10}$ full.

To simplify $2\frac{4}{10}$, keep the whole number the same and simplify the fraction. $\frac{4}{10}$ simplifies to $\frac{2}{5}$

$$2\frac{4}{10} = 2\frac{2}{5}$$

Use Mo's method to simplify:

$3\frac{4}{8}$, $5\frac{9}{21}$, $2\frac{7}{21}$, $\frac{32}{10}$, $\frac{32}{6}$

Simplify Fractions

Reasoning and Problem Solving

Find the total of the fractions.
Give your answer in its simplest form.

$\frac{5}{9} + \frac{1}{9} =$ $\frac{5}{9} + \frac{3}{9} =$ $\frac{5}{9} + \frac{7}{9} =$

Do all the answers need simplifying? Explain why.

$\frac{5}{9} + \frac{1}{9} = \frac{6}{9} = \frac{2}{3}$

$\frac{5}{9} + \frac{3}{9} = \frac{8}{9}$

$\frac{5}{9} + \frac{7}{9} = 1\frac{3}{9} = 1\frac{1}{3}$

$\frac{8}{9}$ does not need simplifying because the HCF of 8 and 9 is 1

Tommy is simplifying $4\frac{12}{16}$

$$4\frac{12}{16} = 1\frac{3}{4}$$

Explain Tommy's mistake.

Tommy has divided the whole number by 4 instead of just simplifying $\frac{12}{16}$ by dividing the numerator and denominator by 4

Sort the fractions into the table.

Simplifies to $\frac{1}{2}$	Simplifies to $\frac{1}{3}$	Simplifies to $\frac{1}{4}$

$\frac{5}{15}$ $\frac{2}{4}$ $\frac{4}{16}$ $\frac{8}{16}$ $\frac{5}{10}$ $\frac{3}{9}$ $\frac{6}{12}$ $\frac{2}{8}$

Can you see any patterns between the numbers in each column?
What is the relationship between the numerators and denominators?
Can you add three more fractions to each column?

Complete the sentence to describe the patterns:
When a fraction is equivalent to _____, the numerator is _______ the denominator.

Simplifies to $\frac{1}{2}$ - $\frac{2}{4}, \frac{8}{16}, \frac{5}{10}, \frac{6}{12}$
Simplifies to $\frac{1}{3}$ - $\frac{5}{15}, \frac{3}{9}$
Simplifies to $\frac{1}{4}$ - $\frac{4}{16}, \frac{2}{8}$

When a fraction is equivalent to a half, the numerator is half the denominator. Children could also discuss the denominator being double the numerator. Repeat for $\frac{1}{3}$ and $\frac{1}{4}$

Improper to Mixed Numbers

Notes and Guidance

Children convert improper fractions to mixed numbers for the first time. An improper fraction is a fraction where the numerator is greater than the denominator. A mixed number is a number consisting of an integer and a proper fraction.

It is important for children to see this process represented visually to allow them to make the connections between the concept and what happens in the abstract.

Mathematical Talk

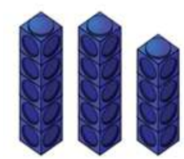
How many parts are there in a whole?

What do you notice happens to the mixed number when the denominator increases and the numerator remains the same?

What happens when the numerator is a multiple of the denominator?

Varied Fluency

- Whitney converts the improper fraction $\frac{14}{5}$ into a mixed number using cubes.
  She groups the cubes into 5s, then has 4 left over.



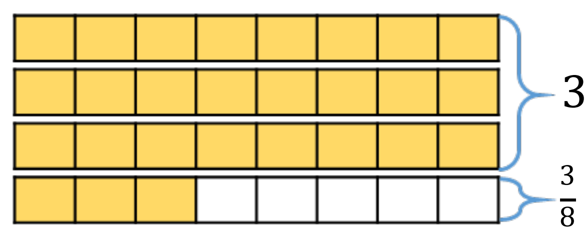
$\frac{5}{5}$ is the same as ☐ $\frac{10}{5}$ is the same as ☐

$\frac{14}{5}$ as a mixed number is ☐ $\frac{☐}{☐}$

Use Whitney's method to convert $\frac{11}{3}$, $\frac{11}{4}$, $\frac{11}{5}$ and $\frac{11}{6}$

- Tommy converts the improper fraction $\frac{27}{8}$ into a mixed number using bar models.

$\frac{27}{8}$



$3\frac{3}{8}$

Use Tommy's method to convert $\frac{25}{8}$, $\frac{27}{6}$, $\frac{18}{7}$ and $\frac{32}{4}$

Improper to Mixed Numbers

Reasoning and Problem Solving

Amir says,

$\frac{28}{3}$ is less than $\frac{37}{5}$ because 28 is less than 37

Do you agree?
Explain why.

Possible answer

I disagree because $\frac{28}{3}$ is equal to $9\frac{1}{3}$ and $\frac{37}{5}$ is equal to $7\frac{2}{5}$

$\frac{37}{5} < \frac{28}{3}$

Spot the mistake

- $\frac{27}{5} = 5\frac{1}{5}$
- $\frac{27}{3} = 8$
- $\frac{27}{4} = 5\frac{7}{4}$
- $\frac{27}{10} = 20\frac{7}{10}$

What mistakes have been made?

Can you find the correct answers?

Correct answers

- $5\frac{2}{5}$ (incorrect number of fifths)
- 9 (incorrect whole)
- $6\frac{3}{4}$ (still have an improper fraction)
- $2\frac{7}{10}$ (incorrect number of wholes)

Mixed Numbers to Improper

Notes and Guidance

Children now convert from mixed numbers to improper fractions using concrete and pictorial methods to understand the abstract method.

Ensure children always write their working alongside the concrete and pictorial representations so they can see the clear links to the abstract.

Mathematical Talk

How many quarters/halves/eighths/fifths are there in a whole?

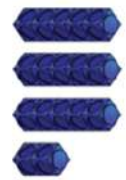
How does multiplication support us in converting from mixed numbers to improper fractions?

Can you explain the steps in converting an improper fraction to a mixed number? Use the vocabulary: numerator, denominator, multiply, add

How could we use the previous bar model to help?

Varied Fluency

Whitney converts $3\frac{2}{5}$ into an improper fraction using cubes.



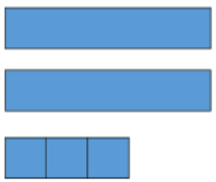
1 whole is equal to □ fifths.

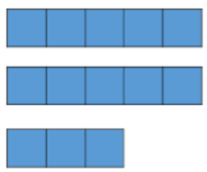
3 wholes are equal to □ fifths.

□ fifths + two fifths = □ fifths

Use Whitney's method to convert $2\frac{2}{3}$, $2\frac{2}{4}$, $2\frac{2}{5}$ and $2\frac{2}{6}$

Jack uses bar models to convert a mixed number into an improper fraction.





$2\frac{3}{5}$ = □ wholes + □ fifths

2 wholes = □ fifths

□ fifths + □ fifths = □ fifths

Use Jack's method to convert $2\frac{1}{6}$, $4\frac{1}{6}$, $4\frac{1}{3}$ and $8\frac{2}{3}$

Mixed Numbers to Improper

Reasoning and Problem Solving

R

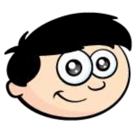
Three children have incorrectly converted $3\frac{2}{5}$ into an improper fraction.

Annie

$3\frac{2}{5} = \frac{6}{15}$

Mo

$3\frac{2}{5} = \frac{15}{5}$



Dexter

$3\frac{2}{5} = \frac{32}{5}$

What mistake has each child made?

Annie has multiplied the numerator and denominator by 3

Mo has multiplied the correctly but then forgotten to add on the extra 2 parts.

Dexter has just placed 3 in front of the numerator.

Fill in the missing numbers.

How many different possibilities can you find for each equation?

$$2\frac{\square}{8} = \frac{\square}{8}$$

$$2\frac{\square}{5} = \frac{\square}{5}$$

Compare the number of possibilities you found.

$2\frac{1}{8} = \frac{17}{8}$ $2\frac{2}{8} = \frac{18}{8}$

$2\frac{3}{8} = \frac{19}{8}$ $2\frac{4}{8} = \frac{20}{8}$

$2\frac{5}{8} = \frac{21}{8}$ $2\frac{6}{8} = \frac{22}{8}$

$2\frac{7}{8} = \frac{23}{8}$

There will be 4 solutions for fifths.

Teacher notes: Encourage children to make generalisations that the number of solutions is one less than the denominator.

Fractions on a Number Line

Notes and Guidance

Children count forwards and backwards in fractions. They compare and order fractions with the same denominator or denominators that are multiples of the same number.
Encourage children to draw extra intervals on the number lines to support them to place the fractions more accurately.
Children use the divisions on the number line to support them in finding the difference between fractions.

Mathematical Talk

Which numbers do I say when I count in eighths and when I count in quarters?

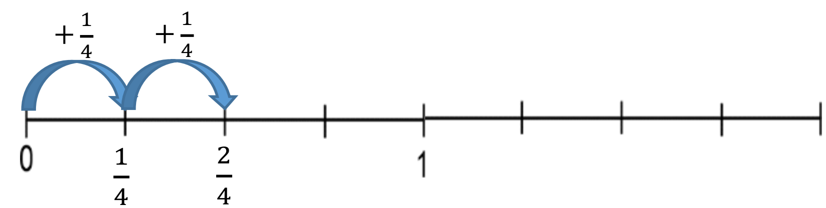
Can you estimate where the fractions will be on the number line?

Can you divide the number line into more intervals to place the fractions more accurately?

How can you find the difference between the fractions?

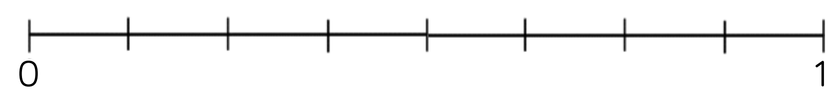
Varied Fluency

Jack is counting in quarters. He writes each number he says on a number line.
Complete Jack's number line.



Can you simplify any of the fractions on the number line?
Can you count forward in eighths? How would the number line change?

Place $\frac{1}{4}$, $\frac{1}{2}$, $\frac{1}{8}$, $\frac{5}{8}$, $\frac{7}{8}$ and $\frac{3}{16}$ on the number line.



Which fractions were the easiest to place?
Which fractions were the hardest to place?
Which fraction is the largest? Which fraction is the smallest?
What is the difference between the largest and smallest fraction?

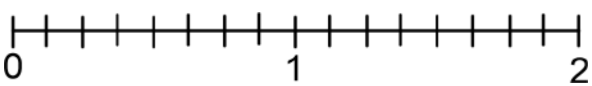
Fractions on a Number Line

Reasoning and Problem Solving

Rosie is counting backwards in fifths.
She starts at $3\frac{2}{5}$ and counts back nine fifths.
What number does Rosie end on?
Show this on a number line.

Rosie ends on $1\frac{3}{5}$

How many ways can you show a difference of one quarter on the number line?



Various answers available.

Plot the sequences on a number line.

$3\frac{1}{2}, 4, 4\frac{1}{2}, 5, 5\frac{1}{2}, 6$

$\frac{13}{4}, \frac{15}{4}, \frac{17}{4}, \frac{19}{4}, \frac{21}{4}, \frac{23}{4}$

$5\frac{5}{8}, 5\frac{1}{8}, 4\frac{5}{8}, 4\frac{1}{8}, 3\frac{5}{8}, 3\frac{1}{8}$

$3\frac{1}{8}, 3\frac{3}{8}, 3\frac{5}{8}, 3\frac{7}{8}, 4\frac{1}{8}, 4\frac{3}{8}$

Which sequence is the odd one out? Explain why.

Can you think of a reason why each of the sequences could be the odd one out?

Children may choose different sequences for different reasons.
First sequence: the only one containing 6 or it is the only one containing whole numbers.
Second sequence: only one using improper fractions
Third sequence: the only one going backwards.
Fourth sequence: only one not counting in halves.

Compare & Order (Denominator)

Notes and Guidance

Children use their knowledge of equivalent fractions to compare fractions where the denominators are not multiples of the same number.
They find the lowest common multiple of the denominators in order to find equivalent fractions with the same denominators.
Children then compare the numerators to find the larger or smaller fraction. Encourage children to also use their number sense to visualise the size of the fractions before converting.

Mathematical Talk

When I know the lowest common multiple, how do I know what to multiply the numerator and denominator by to find the correct equivalent fraction?
How is comparing mixed numbers different to comparing proper fractions? Do I need to compare the whole numbers? Why? If the whole numbers are the same, what do I do?
Can you plot the fractions on a number line to estimate which is the smallest? Which fractions are larger/smaller than a half? How does this help me order the fractions?

Varied Fluency

Use the bar models to compare $\frac{3}{4}$ and $\frac{2}{3}$

____ is greater than ____

____ is less than ____

Dora is comparing $\frac{5}{6}$ and $\frac{3}{4}$ by finding the lowest common multiple of the denominators.

Multiples of 6: 6, 12, 18, 24
Multiples of 4: 4, 8, 12, 16,
12 is the LCM of 4 and 6

$$\frac{5}{6} = \frac{10}{12} \qquad \frac{3}{4} = \frac{9}{12}$$

$$\frac{10}{12} > \frac{9}{12}$$

Use Dora's method to compare the fractions.

$\frac{4}{5}$ ◯ $\frac{3}{4}$ $\frac{3}{5}$ ◯ $\frac{4}{7}$ $\frac{3}{4}$ ◯ $\frac{7}{10}$ $2\frac{2}{5}$ ◯ $2\frac{3}{8}$

Order the fractions in descending order.

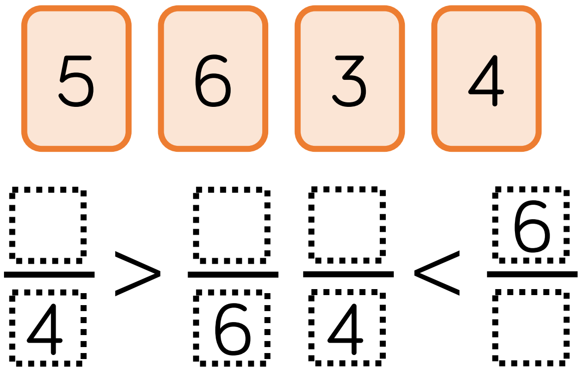
$$\frac{3}{8}, \frac{11}{20}, \frac{1}{2}, \frac{2}{5}, \frac{3}{4}, \frac{7}{10}$$

Which fraction is the greatest?
Which fraction is the smallest?

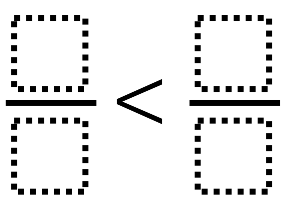
Compare & Order (Denominator)

Reasoning and Problem Solving

Use the digit cards to complete the statements.



Find three examples of ways you could complete the statement.



Can one of your ways include an improper fraction?

$\frac{5}{4} > \frac{3}{6}$

$\frac{3}{4} < \frac{6}{5}$ or $\frac{5}{4} < \frac{6}{3}$

$\frac{3}{5} < \frac{6}{4}$

$\frac{3}{4} < \frac{6}{5}$

$\frac{4}{5} < \frac{6}{3}$

More answers available.

Teddy is comparing $\frac{3}{8}$ and $\frac{5}{12}$

To find the lowest common multiple, I will multiply 8 and 12 together.
$8 \times 12 = 96$
I will use a common denominator of 96

Is Teddy correct?
Explain why.

Teddy is incorrect because the LCM of 8 and 12 is 24. 96 is a common multiple so he would still compare the fractions correctly but it is not the most efficient method.

Compare & Order (Numerator)

Notes and Guidance

Building on their prior knowledge of comparing unit fractions, children look at comparing fractions by finding a common numerator. They focus on the idea that when the numerators are the same, the larger the denominator, the smaller the fraction.
Children consider the most efficient method when comparing fractions and decide whether to find common numerators or common denominators.

Mathematical Talk

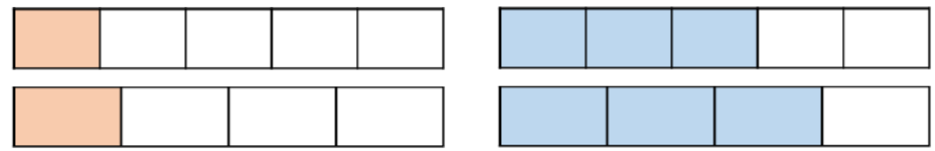
What's the same and what's different about the fractions on the bar models? How can we compare them? Can you use the words greatest and smallest to complete the sentences?

Do you need to change one or both numerators? Why?

How can you decide whether to find a common numerator or denominator?

Varied Fluency

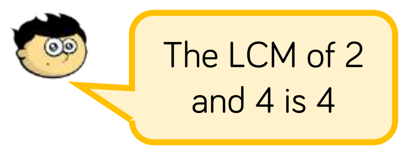
Compare the fractions.



$\frac{1}{5}$ ◯ $\frac{1}{4}$ $\frac{1}{5}$ ◯ $\frac{3}{5}$ $\frac{1}{4}$ ◯ $\frac{3}{4}$ $\frac{3}{5}$ ◯ $\frac{3}{4}$

When the denominators are the same, the ______ the numerator, the ______ the fraction.
When the numerators are the same, the ______ the denominator, the ______ the fraction.

Jack is comparing $\frac{2}{5}$ and $\frac{4}{7}$ by finding the LCM of the numerators.



$\frac{2}{5} = \frac{4}{10}$ $\frac{4}{10} < \frac{4}{7}$

Use Jack's method to compare the fractions.

$\frac{3}{5}$ ◯ $\frac{12}{17}$ $\frac{6}{11}$ ◯ $\frac{3}{5}$ $\frac{5}{9}$ ◯ $\frac{4}{7}$ $\frac{8}{5}$ ◯ $\frac{12}{7}$

Compare & Order (Numerator)

Reasoning and Problem Solving

Mo is comparing the fractions $\frac{3}{7}$ and $\frac{6}{11}$ He wants to find a common denominator. Explain whether you think this is the most effective strategy.	This is not the most effective strategy because both denominators are prime. He could find a common numerator by changing $\frac{3}{7}$ into $\frac{6}{14}$ and comparing them by using the rule 'when the numerator is the same, the smaller the denominator, the bigger the fraction' $\frac{6}{11}$ is bigger.
Two different pieces of wood have had a fraction chopped off. Here are the pieces now, with the fraction that is left.  $\frac{5}{6}$ $\frac{3}{4}$ Which piece of wood was the longest to begin with? Explain your answer. Can you explain your method?	The second piece was longer because $\frac{1}{4}$ is greater than $\frac{1}{6}$. Children can explain their methods and how they compared one quarter and one sixth.

Add & Subtract Fractions (1)

Notes and Guidance

Children add and subtract fractions within 1 where the denominators are multiples of the same number.
Encourage children to find the lowest common multiple in order to find a common denominator. Ensure children are confident with the understanding of adding and subtracting fractions with the same denominator. Bar models can support this, showing children that the denominators stay the same whilst the numerators are added or subtracted.

Mathematical Talk

If the denominators are different, when we are adding or subtracting fractions, what do we need to do? Why?

How does finding the lowest common multiple help to find a common denominator?

Can you use a bar model to represent Eva's tin of paint?
On which day did Eva use the most paint? On which day did Eva use the least paint? How much more paint did Eva use on Friday than Saturday?

Varied Fluency

- Whitney is calculating $\frac{5}{8} + \frac{3}{16}$
  She finds the lowest common multiple of 8 and 16 to find a common denominator.

  LCM of 8 and 16 is 16 $\quad \frac{5}{8} = \frac{10}{16} \quad \frac{10}{16} + \frac{3}{16} = \frac{13}{16}$

  Use this method to calculate:

  $\frac{1}{3} + \frac{2}{9} =$ $\quad \frac{3}{7} + \frac{7}{21} =$ $\quad \frac{8}{15} + \frac{1}{5} =$ $\quad \frac{3}{16} + \frac{3}{8} + \frac{1}{4} =$

- Find a common denominator for each pair of fractions by using the lowest common multiple. Subtract the smaller fraction from the larger fraction in each pair.

  $\frac{3}{4}, \frac{5}{8}$ $\quad \frac{7}{12}, \frac{1}{3}$ $\quad \frac{11}{16}, \frac{3}{4}$ $\quad \frac{14}{15}, \frac{2}{5}$ $\quad \frac{8}{9}, \frac{1}{3}$

- Eva has a full tin of paint. She uses $\frac{1}{3}$ of the tin on Friday, $\frac{1}{21}$ on Saturday and $\frac{2}{7}$ on Sunday. How much paint does she have left?

White Rose Maths

Add & Subtract Fractions (1)

Reasoning and Problem Solving

Use the same digit in both boxes to complete the calculation.
Is there more than one way to do it?

$$\frac{\square}{20} + \frac{1}{\square} = \frac{9}{20}$$

$\frac{4}{20} + \frac{1}{4} = \frac{9}{20}$

$\frac{5}{20} + \frac{1}{5} = \frac{9}{20}$

Dexter subtracted $\frac{3}{5}$ from a fraction and his answer was $\frac{8}{45}$

What fraction did he subtract $\frac{3}{5}$ from?

Give your answer in its simplest form.

$\frac{8}{45} + \frac{3}{5} = \frac{8}{45} + \frac{27}{45}$

$\frac{8}{45} + \frac{27}{45} = \frac{35}{45} = \frac{7}{9}$

Dexter subtracted $\frac{3}{5}$ from $\frac{7}{9}$

Alex is adding fractions.

$$\frac{3}{5} + \frac{1}{15} = \frac{4}{20} = \frac{1}{5}$$

Do you agree with her?

Explain your answer.

Alex is wrong because she has added the numerators and the denominators rather than finding a common denominator.

It should be

$\frac{9}{15} + \frac{1}{15} = \frac{10}{15} = \frac{2}{3}$

Add & Subtract Fractions (2)

Notes and Guidance

Children add and subtract fractions where the denominators are not multiples of the same number. They continue to find the lowest common multiple, but now need to find equivalent fractions for both fractions in the calculation to find a common denominator.
When the denominators are not multiples of the same number, support children to notice that we need to multiply the denominators together in order to find the LCM.

Mathematical Talk

What is the same about all the subtractions? ($\frac{3}{4}$)

What do you notice about the LCM of all the denominators?

Which of the subtractions has the biggest difference? Explain how you know. Can you order the differences in ascending order?

How can we find the LCM of three numbers? Do we multiply them together? Is 120 the LCM of 4, 5 and 6?

Varied Fluency

- Amir is calculating $\frac{7}{9} - \frac{1}{2}$
He finds the lowest common multiple of 9 and 2
LCM of 9 and 2 is 18

$$\frac{7}{9} - \frac{1}{2} = \frac{14}{18} - \frac{9}{18} = \frac{5}{18}$$

Use this method to calculate:

$\frac{3}{4} - \frac{1}{3} =$ $\frac{3}{4} - \frac{3}{5} =$ $\frac{3}{4} - \frac{2}{7} =$ $\frac{3}{4} - \frac{7}{11} =$

- Eva has a bag of carrots weighing $\frac{3}{4}$ kg and a bag of potatoes weighing $\frac{2}{5}$ kg. She is calculating how much they weigh altogether.

$$\frac{3}{4} + \frac{2}{5} = \frac{15}{20} + \frac{8}{20} = \frac{23}{20} = 1\frac{3}{20} \text{ kg}$$

Use this method to calculate:

$\frac{1}{4} + \frac{2}{5} =$ $\frac{7}{8} + \frac{1}{3} =$ $\frac{5}{6} + \frac{5}{7} =$ $\frac{13}{20} + \frac{2}{3} =$

- On Friday, Ron walks $\frac{5}{6}$ km to school, $\frac{3}{4}$ km to the shops and $\frac{4}{5}$ km home. How far does he walk altogether?

Add & Subtract Fractions (2)

Reasoning and Problem Solving

A car is travelling from Halifax to Brighton. In the morning, it completes $\frac{2}{3}$ of the journey. In the afternoon, it completes $\frac{1}{5}$ of the journey. What fraction of the journey has been travelled altogether? What fraction of the journey is left to travel?	The car has travelled $\frac{13}{15}$ of the journey altogether. There is $\frac{2}{15}$ of the journey left to travel.
If the journey is 270 miles, how far did the car travel in the morning? How far did the car travel in the afternoon? How far does the car have left to travel? 	The car travelled 180 miles in the morning. The car travelled 54 miles in the afternoon. The car has 36 miles left to travel.

Mr and Mrs Rose and knitting scarves. Mr Rose's scarf is $\frac{5}{9}$ m long. Mrs Rose's scarf is $\frac{1}{5}$ m longer than Mr Rose's scarf. How long is Mrs Rose's scarf? How long are both the scarves altogether?	Mrs Rose's scarf is $\frac{34}{45}$ m long. Both scarves together are $1\frac{14}{45}$ m long.
Fill in the boxes to make the calculation correct. $1\frac{\square}{10} = \frac{3}{\square} + \frac{\square}{10}$	Various answers available. E.g. $1\frac{1}{10} = \frac{3}{5} + \frac{5}{10}$

Add Mixed Numbers

Notes and Guidance

Children move on to adding two fractions where one or both are mixed numbers or improper fractions.

They will use a method of adding the wholes and then adding the parts. Children will record their answer in its simplest form.

Children can still draw models to represent adding fractions.

Mathematical Talk

How can we partition these mixed numbers into whole numbers and fractions?

What will the wholes total? Can I add the fractions straight away?

What will these mixed numbers be as improper fractions?

If I have an improper fraction in the question, should I change it to a mixed number first? Why?

Varied Fluency

$1\frac{1}{3} + 2\frac{1}{6} = 3 + \frac{3}{6} = 3\frac{3}{6}$ or $3\frac{1}{2}$

$1 + 2 = 3$

$\frac{1}{3} + \frac{1}{6} = \frac{2}{6} + \frac{1}{6} = \frac{3}{6}$

Add the fractions by adding the whole first and then the fractions. Give your answer in its simplest form.

$3\frac{1}{4} + 2\frac{3}{8}$ $\quad 4\frac{1}{9} + 3\frac{2}{3}$ $\quad 2\frac{5}{12} + 2\frac{1}{3}$

$1\frac{3}{4} + 2\frac{1}{8} = \frac{7}{4} + \frac{17}{8} = \frac{14}{8} + \frac{17}{8} = \frac{31}{8} = 3\frac{7}{8}$

Add the fractions by converting them to improper fractions.

$1\frac{1}{4} + 2\frac{5}{12}$ $\quad 2\frac{1}{9} + 1\frac{1}{3}$ $\quad 2\frac{1}{6} + 2\frac{2}{3}$

Add these fractions.

$4\frac{7}{9} + 2\frac{1}{3}$ $\quad \frac{17}{6} + 1\frac{1}{3}$ $\quad \frac{15}{8} + 2\frac{1}{4}$

How do they differ from previous examples?

Add Mixed Numbers

Reasoning and Problem Solving

Jack and Whitney have some juice. Jack drinks $2\frac{1}{4}$ litres and Whitney drinks $2\frac{5}{12}$ litres. How much do they drink altogether? Complete this using two different methods. Which method do you think is more efficient? Why?	They drink $4\frac{2}{3}$ litres altogether. Encourage children to justify which method they prefer and why. Ensure children discuss which method is more or less efficient.	Fill in the missing numbers. $4\frac{5}{6} + \square\frac{\square}{\square} = 10\frac{1}{3}$	$5\frac{3}{6}$ or $5\frac{1}{2}$

White Rose Maths

Add Fractions

Notes and Guidance

Children explore adding mixed numbers. They look at different methods depending on whether the fractions total more than one. They add fractions with any denominators, building on their understanding from the previous steps.
Encourage children to draw bar models to support them in considering whether the fractions will cross the whole. They continue to simplify answers and convert between improper fractions and whole numbers when calculating.

Mathematical Talk

How many wholes are there altogether?

Can you find the LCM of the denominators to find a common denominator?

Do you prefer Tommy or Whitney's method? Why?

Does Tommy's method work when the fractions add to more than one? How could we adapt his method?

Does Whitney's method work effectively when there are large whole numbers?

Varied Fluency

Tommy is adding mixed numbers. He adds the wholes and then adds the fractions. Then, Tommy simplifies his answer.

$$1\frac{1}{2} + 2\frac{1}{6} = 1\frac{3}{6} + 2\frac{1}{6} = 3\frac{4}{6} = 3\frac{2}{3}$$

Use Tommy's method to add the fractions.

$3\frac{1}{2} + 2\frac{3}{8} =$ $\quad$ $34\frac{1}{9} + 5\frac{2}{5} =$ $\quad$ $12\frac{5}{12} + 2\frac{1}{7} =$

Whitney is also adding mixed numbers. She converts them to improper fractions, adds them, and then converts them back to a mixed number.

$$1\frac{1}{2} + 2\frac{1}{6} = \frac{3}{2} + \frac{13}{6} = \frac{9}{6} + \frac{13}{6} = \frac{22}{6} = 3\frac{4}{6} = 3\frac{2}{3}$$

Use Whitney's method to add the fractions.

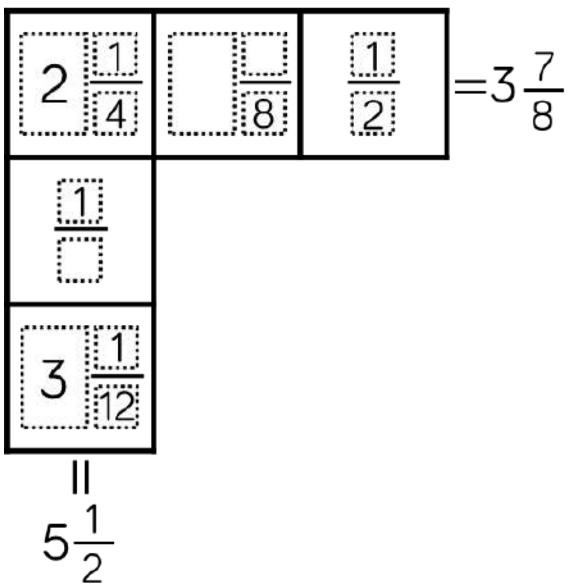
$3\frac{1}{2} + 2\frac{3}{8}$ $\quad$ $2\frac{1}{9} + 2\frac{2}{5}$ $\quad$ $2\frac{7}{9} + 2\frac{2}{5}$ $\quad$ $4\frac{3}{4} + 3\frac{11}{15}$

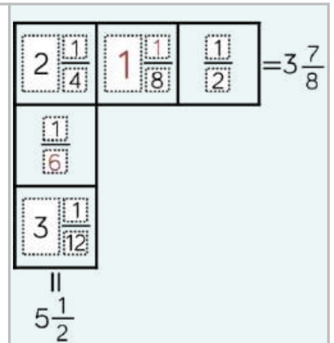
Jug A has $2\frac{3}{4}$ litres of juice in it. Jug B has $3\frac{4}{5}$ litres of juice in it.
How much juice is there in Jug A and Jug B altogether?

Add Fractions

Reasoning and Problem Solving

Each row and column adds up to make the total at the end.
Use this information to complete the diagram.





Dora is baking muffins.
She uses $2\frac{1}{2}$ kg of flour, $1\frac{3}{5}$ kg of sugar and $1\frac{1}{4}$ kg of butter.

How much flour, sugar and butter does she use altogether?

How much more flour does she use than butter?

How much less butter does she use than sugar?

Dora uses $5\frac{7}{20}$ kg of flour, sugar and butter altogether.

Dora uses $1\frac{1}{4}$ kg more flour than butter.

Dora uses $\frac{7}{20}$ kg less butter than sugar.

Subtract Mixed Numbers (1)

Notes and Guidance

Children apply their understanding of subtracting fractions where one denominator is a multiple of the other to subtract proper fractions from mixed numbers.

They continue to use models and number lines to support their understanding.

Mathematical Talk

Which fraction is the greatest? How do you know?

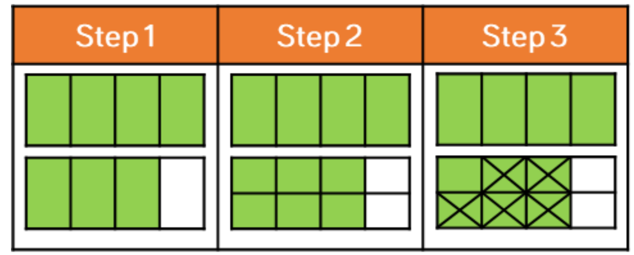
If the denominators are different, what can we do?

Can you simplify your answer?

Which method do you prefer when subtracting fractions: taking away or finding the difference?

Varied Fluency

R



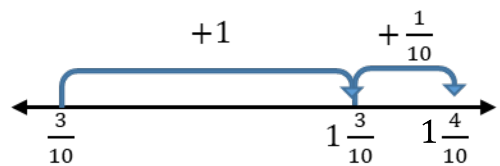
$1\frac{3}{4} - \frac{5}{8} = 1\frac{1}{8}$

Use this method to help you solve:

$2\frac{3}{5} - \frac{3}{10}$ $1\frac{2}{3} - \frac{1}{6}$ $1\frac{5}{6} - \frac{7}{12}$

Use a number line to find the difference between $1\frac{2}{5}$ and $\frac{3}{10}$

$1\frac{2}{5} = 1\frac{4}{10}$



Use a number line to find the difference between:

$3\frac{5}{6}$ and $\frac{1}{12}$ $5\frac{5}{7}$ and $\frac{3}{14}$ $2\frac{7}{9}$ and $\frac{11}{18}$

Solve:

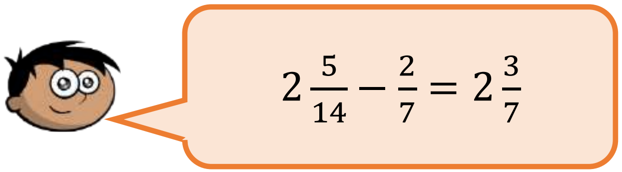
$1\frac{2}{3} - \frac{5}{6}$ $1\frac{3}{4} - \frac{7}{8}$ $2\frac{3}{8} - \frac{11}{16}$

Subtract Mixed Numbers (1)

Reasoning and Problem Solving

Amir is attempting to solve $2\frac{5}{14} - \frac{2}{7}$

Here is his working out:

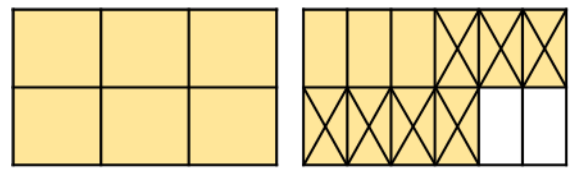


Do you agree with Amir?
Explain your answer.

Possible answer:

Amir is wrong because he hasn't found a common denominator when subtracting the fractions he has just subtracted the numerators and the denominators. The correct answer is $2\frac{1}{14}$

Here is Rosie's method.
What is the calculation?



Can you find more than one answer?
Why is there more than one answer?

The calculation could be $1\frac{5}{6} - \frac{7}{12}$ or $1\frac{10}{12} - \frac{7}{12}$

There is more than one answer because five sixths and ten twelfths are equivalent. Children should be encouraged to write the question as $1\frac{5}{6} - \frac{7}{12}$ so that all fractions are in their simplest form.

Subtract Fractions

Notes and Guidance

Children subtract mixed numbers. They explore different methods including exchanging wholes for fractions and subtracting the wholes and fractions separately and converting the mixed number to an improper fraction.
Encourage children to consider which method is the most efficient depending on the fractions they are subtracting. Bar models can support to help children to visualise the subtraction and understand the procedure.

Mathematical Talk

How many eighths can we exchange for one whole?

What is the same about the first set of subtractions?

What is different about the subtractions? (How does this affect the subtraction?

Do you prefer Annie's or Amir's method? Why?

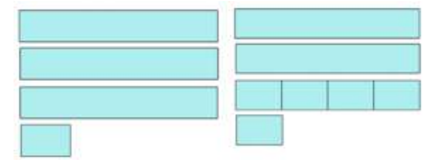
Look at Amir's calculation, what do you notice about the relationship between $3\frac{2}{5}$ and $1\frac{7}{10}$? ($3\frac{2}{5}$ is double $1\frac{7}{10}$)

Varied Fluency

Annie is calculating $3\frac{1}{4} - 1\frac{3}{4}$

I can't subtract the wholes and fractions separately because $\frac{1}{4}$ is less than $\frac{3}{4}$. I will exchange 1 whole for 4 quarters. $3\frac{1}{4} = 2\frac{5}{4}$

$3\frac{1}{4} - 1\frac{3}{4} = 2\frac{5}{4} - 1\frac{3}{4} = 1\frac{2}{4} = 1\frac{1}{2}$



Use Annie's method to calculate:

$3\frac{1}{8} - 1\frac{3}{8} =$ $3\frac{1}{8} - 1\frac{1}{2} =$ $3\frac{1}{8} - 1\frac{1}{5} =$ $3\frac{1}{8} - 1\frac{3}{5} =$

Amir is calculating $3\frac{2}{5} - 1\frac{7}{10}$

He converts the mixed numbers to improper fractions to subtract them.

$3\frac{2}{5} - 1\frac{7}{10} = \frac{17}{5} - \frac{17}{10} = \frac{34}{10} - \frac{17}{10} = \frac{17}{10} = 1\frac{7}{10}$

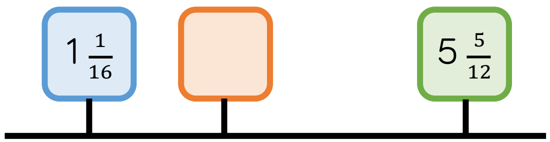
Convert the mixed numbers to improper fractions to calculate:

$4\frac{4}{5} - 1\frac{9}{10} =$ $2\frac{1}{7} - 1\frac{1}{3} =$ $3\frac{5}{12} - 1\frac{7}{9} =$ $3\frac{5}{11} - 1\frac{4}{5} =$

Subtract Fractions

Reasoning and Problem Solving

A blue, orange and green box are on a number line.



The number in the green box is $3\frac{2}{3}$ more than the orange box.

The number in the orange box is: $\square\frac{\square}{\square}$

The number in the orange box is $\frac{\square}{\square}$ greater than the number in the blue box.

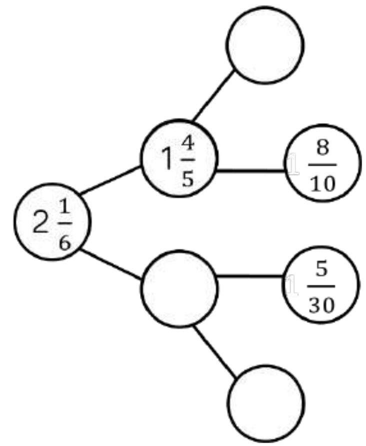
$5\frac{5}{12} - 3\frac{2}{3} = 1\frac{9}{12}$

The orange box is $1\frac{3}{4}$

$1\frac{3}{4} - 1\frac{1}{16} = \frac{11}{16}$

The orange box is $\frac{11}{16}$ greater than the blue box.

Complete the part-whole model.



$2\frac{1}{6} - 1\frac{4}{5} = 1\frac{11}{30}$

$1\frac{4}{5} - \frac{8}{10} = 1$

$1\frac{11}{30} - \frac{5}{30} = 1\frac{6}{30} = 1\frac{1}{5}$

Jack is calculating $4\frac{2}{7} - 2\frac{6}{7}$

He adds $\frac{1}{7}$ to both numbers.

$4\frac{2}{7} - 2\frac{6}{7} = 4\frac{3}{7} - 3$

so the answer is $1\frac{3}{7}$

Explain why Jack is correct.

Jack has increased both mixed numbers by $\frac{1}{7}$ so the difference has remained constant.

Mixed Addition & Subtraction

Notes and Guidance

Children solve problems that involve adding and subtracting fractions and mixed numbers. Encourage children to consider the most efficient method of adding and subtracting fractions and to simplify their answers when possible.
Children can use bar models to represent the problems and support them in deciding whether they need to add or subtract. They can share their different methods to gain a flexible approach to calculating with fractions.

Mathematical Talk

Can you draw a bar model to represent the problem? Do we need to add or subtract the fractions?

How do I know if my answer is simplified fully?

What is the lowest common multiple of the denominators?

How can I calculate the area covered by each vegetable? If you know the area for carrots and cabbages, how can you work out the area for potatoes? Can you think of 2 different ways?

Varied Fluency

- Alex has 5 bags of sweets.

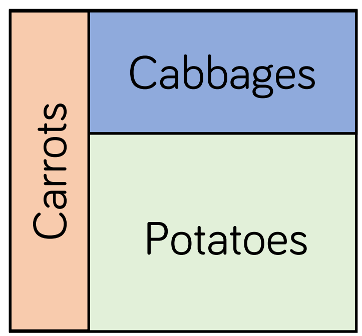
  On Monday she eats $\frac{2}{3}$ of a bag and gives $\frac{4}{5}$ of a bag to her friend.

  On Tuesday she eats $1\frac{1}{3}$ bags and gives $\frac{2}{5}$ of a bag to her friend.

  What fraction of her sweets does Alex have left?

  Give your answer in its simplest form.

- Here is a vegetable patch. $\frac{1}{5}$ of the patch is for carrots. $\frac{3}{8}$ of the patch is for cabbages.



  What fraction of the patch is for carrots and cabbages altogether?
  What fraction of the patch is for potatoes?
  What fraction more of the patch is for potatoes than cabbages?
  Give your answers in their simplest form.

  The vegetable patch has an area of 80 m^2
  What is the area covered by each vegetable?

Mixed Addition & Subtraction

Reasoning and Problem Solving

The mass of Annie's suitcase is $29\frac{1}{2}$ kg.

Teddy's suitcase is $2\frac{1}{5}$ kg lighter than Annie's.

How much does Teddy's suitcase weigh?

How much do the suitcases weigh altogether?

There is a weight allowance of 32 kg per suitcase.

How much below the weight allowance are Annie and Teddy?

Teddy's suitcase weighs $27\frac{3}{10}$ kg

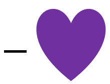
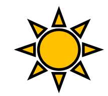
The suitcases weigh $56\frac{4}{5}$ kg altogether.

Annie is $2\frac{1}{2}$ kg under the weight allowance.

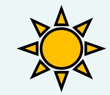
Teddy is $4\frac{7}{10}$ kg under the weight allowance.

Find the value of the 

 $+ 3\frac{4}{9} = 6\frac{1}{3}$

$8\frac{1}{10} -$  $=$ 

The value of the  is $2\frac{8}{9}$

The value of the  is $5\frac{19}{90}$

Multiply Fractions by Integers

Notes and Guidance

Children multiply fractions and mixed numbers by integers. They use diagrams to highlight the link between multiplication and repeated addition. This supports the children in understanding why the denominator stays the same and we multiply the numerator.
When multiplying mixed numbers, children partition into wholes and parts to multiply more efficiently. They compare this method with multiplying improper fractions.

Mathematical Talk

How is multiplying fractions similar to adding fractions?

How does partitioning the mixed number into wholes and fractions support us to multiply?

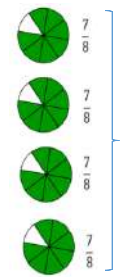
Do you prefer partitioning the mixed number or converting it to an improper fraction to multiply? Why?

Does it matter if the integer is first or second in the multiplication sentence? Why?

Varied Fluency

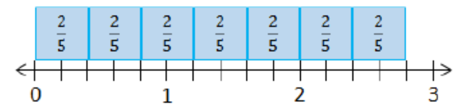
- Complete:

$4 \times \frac{7}{8}$



$3 \times \frac{2}{3}$

$\frac{2}{5} \times 7$

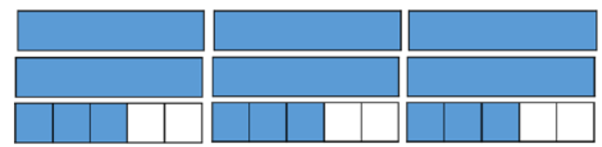


- Eva partitions $2\frac{3}{5}$ to help her to calculate $2\frac{3}{5} \times 3$

$2 \times 3 = 6$

$\frac{3}{5} \times 3 = \frac{9}{5} = 1\frac{4}{5}$

$6 + 1\frac{4}{5} = 7\frac{4}{5}$



Use Eva's method to calculate:

$2\frac{5}{6} \times 3$ $1\frac{3}{7} \times 5$ $2\frac{2}{3} \times 3$ $4 \times 1\frac{1}{6}$

- Convert the mixed number to an improper fraction to multiply.

$$2\frac{3}{5} \times 3 = \frac{13}{5} \times 3 = \frac{39}{5} = 7\frac{4}{5}$$

Use this method to calculate:

$3 \times 2\frac{2}{5}$ $1\frac{5}{7} \times 3$ $2 \times 1\frac{3}{4}$ $2 \times 1\frac{1}{6}$

Multiply Fractions by Integers

Reasoning and Problem Solving

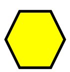
There are 9 lamp posts on a road. There is $4\frac{3}{8}$ of a metre between each lamp post.

What is the distance between the first and last lamp post?

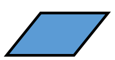
$8 \times 4\frac{3}{8} = 8 \times \frac{35}{8}$

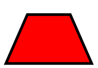
$= \frac{280}{8} = 35$

The distance between the first and last lamp post is 35 metres.

Use pattern blocks, if  is equal to 1 whole, work out what fraction the other shapes represent.
Use this to calculate the multiplications.
Give your answers in their simplest form.

 $\times 5 =$

 $\times 5 =$

 $\times 5 =$

 $\times 5 = \frac{5}{6}$

 $\times 5 = \frac{5}{3} = 1\frac{2}{3}$

 $\times 5 = \frac{5}{2} = 2\frac{1}{2}$

Eva and Amir both work on a homework project.

Eva


I spent $4\frac{1}{4}$ hours a week for 4 weeks doing my project.

I spent $2\frac{3}{4}$ hours a week for 5 weeks doing my project.


Amir

Who spent the most time on their project?

Explain your reasoning.

$4 \times 4\frac{1}{4} = \frac{68}{4}$

$= 17$ hours

$5 \times 2\frac{3}{4} = \frac{55}{4}$

$= 13\frac{3}{4}$ hours

Eva spent $3\frac{1}{4}$ hours longer on her project than Amir did.

Multiply Fractions by Fractions

Notes and Guidance

Children use concrete and pictorial representations to support them to multiply fractions. Support children in understanding the link between multiplying fractions and finding fractions of an amount: $\frac{1}{3} \times \frac{1}{2}$ is the same as $\frac{1}{3}$ of $\frac{1}{2}$

Encourage children to spot the patterns of what is happening in the multiplication, to support them in unpicking the procedure of multiplying fractions by multiplying the numerators and multiplying the denominators.

Mathematical Talk

Could you use folding paper to calculate $\frac{2}{3} \times \frac{1}{2}$? How? Use a piece of paper to model this to a friend.

How are the diagrams similar to folding paper? Which do you find more efficient?

What do you notice about the product of the fractions you have multiplied? What is the procedure to multiply fractions?

Does multiplying two numbers always give you a larger product? Explain why.

Varied Fluency

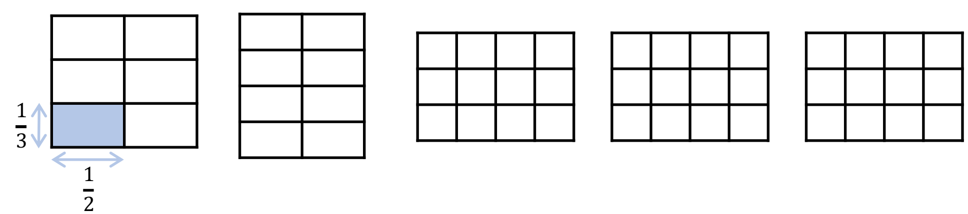
- Dexter is calculating $\frac{1}{3} \times \frac{1}{2}$ by folding paper. He folds a piece of paper in half. He then folds the half into thirds.He shades the fraction of paper he has created. When he opens it up he finds he has shaded $\frac{1}{6}$ of the whole piece of paper.

$\frac{1}{3} \times \frac{1}{2}$ means $\frac{1}{3}$ of a half. Folding half the paper into three equal parts showed me that $\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$

Represent and calculate the multiplications by folding paper.

$\frac{1}{4} \times \frac{1}{2} =$ $\frac{1}{4} \times \frac{1}{3} =$ $\frac{1}{4} \times \frac{1}{4} =$

- Alex is drawing diagrams to represent multiplying fractions.



Shade the diagrams to calculate:

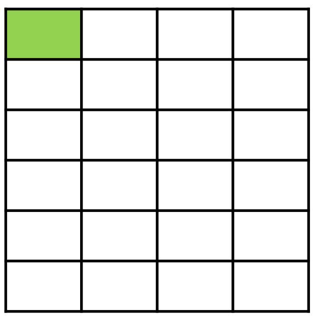
$\frac{1}{3} \times \frac{1}{2} =$ $\frac{1}{4} \times \frac{1}{2} =$ $\frac{1}{3} \times \frac{1}{4} =$ $\frac{2}{3} \times \frac{1}{4} =$ $\frac{2}{3} \times \frac{3}{4} =$

Write your answers in their simplest form.

Multiply Fractions by Fractions

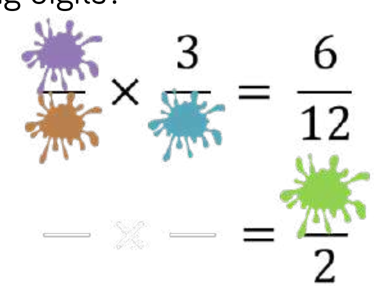
Reasoning and Problem Solving

The shaded square in the grid below is the answer to a multiplying fractions question.
What was the question?



$\frac{1}{6} \times \frac{1}{4}$

How many ways can you complete the missing digits?



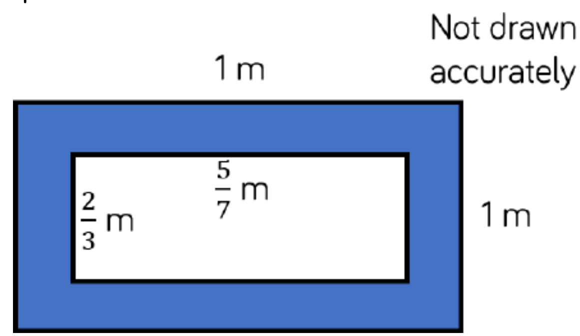
Possible answers:

$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12} = \frac{1}{2}$

$\frac{2}{2} \times \frac{3}{6} = \frac{6}{12} = \frac{1}{2}$

Children could also use improper fractions.

Find the area of the shaded part of the shape.



$1 \times 1 = 1$

$\frac{2}{3} \times \frac{5}{7} = \frac{10}{21}$

$1 - \frac{10}{21} = \frac{11}{21}$

The shaded area is $\frac{11}{21}$ m^2.

Alex says,

$\frac{1}{4} \times \frac{1}{2}$ is the same as $\frac{1}{2}$ of a quarter.

Do you agree?
Explain why.

Alex is correct. Multiplication is commutative so $\frac{1}{4} \times \frac{1}{2}$ is the same as $\frac{1}{2}$ of a quarter or $\frac{1}{4}$ of a half.

Divide Fractions by Integers (1)

Notes and Guidance

Children are introduced to dividing fractions by integers for the first time. They focus on dividing fractions where the numerator is a multiple of the integer they are dividing by. Encourage children to spot the pattern that the denominator stays the same and the numerator is divided by the integer
Children link dividing fractions to multiplying by unit fractions. Use the diagrams children drew for multiplying fractions to discuss how and why the calculations are similar.

Mathematical Talk

How could you represent this fraction?
Is the numerator divisible by the integer?

Why doesn't the denominator change?

What pattern can you see when dividing elevenths?

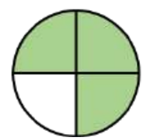
How can we use the pattern to help us to calculate a mixed number by an integer? Can you convert it to an improper fraction?

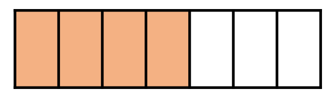
Varied Fluency

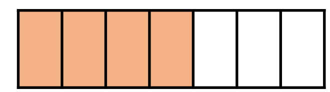
- Dexter has $\frac{2}{5}$ of a chocolate bar. He shares it with his friend. What fraction of the chocolate bar do they each get?

- Use the diagrams to help you calculate.

$\frac{3}{4} \div 3 =$ $\frac{4}{7} \div 4 =$ $\frac{4}{7} \div 2 =$







- Calculate.

$\frac{1}{11} \div 1 =$ $\frac{2}{11} \div 2 =$ $\frac{3}{11} \div 3 =$ $\frac{4}{11} \div 4 =$

$\frac{2}{11} \div 2 =$ $\frac{4}{11} \div 2 =$ $\frac{6}{11} \div 2 =$ $\frac{8}{11} \div 2 =$

$\frac{3}{11} \div 3 =$ $\frac{6}{11} \div 3 =$ $\frac{9}{11} \div 3 =$ $1\frac{1}{11} \div 3 =$

Divide Fractions by Integers (1)

Reasoning and Problem Solving

Tommy says,

Dividing by 2 is the same as finding half of a number so $\frac{4}{11} \div 2$ is the same as $\frac{1}{2} \times \frac{4}{11}$

Do you agree?
Explain why.

Tommy is correct.
It may help children to understand this by reinforcing that $\frac{1}{2} \times \frac{4}{11}$ is the same as $\frac{1}{2}$ of $\frac{4}{11}$

Match the equivalent calculations.

$\frac{1}{4} \times \frac{12}{13}$	$\frac{12}{13} \div 2$
$\frac{1}{6} \times \frac{12}{13}$	$\frac{12}{13} \div 6$
$\frac{1}{2} \times \frac{12}{13}$	$\frac{12}{13} \div 4$
$\frac{1}{3} \times \frac{12}{13}$	$\frac{12}{13} \div 3$

$\frac{1}{4} \times \frac{12}{13} = \frac{12}{13} \div 4$

$\frac{1}{6} \times \frac{12}{13} = \frac{12}{13} \div 6$

$\frac{1}{2} \times \frac{12}{13} = \frac{12}{13} \div 2$

$\frac{1}{3} \times \frac{12}{13} = \frac{12}{13} \div 3$

Complete the missing integers.

$\frac{15}{16} \div \square = \frac{5}{16}$

$\frac{15}{16} \div \square = \frac{3}{16}$

$\frac{20}{23} \div \square = \frac{4}{23}$

$\frac{20}{23} \div \square = \frac{5}{23}$

3
5
5
4

Rosie walks for $\frac{3}{4}$ of an hour over 3 days.
She walks for the same amount of time each day.
How many minutes does Rosie walk each day?

Rosie walks for $\frac{1}{4}$ of an hour each day.
She walks for 15 minutes each day.

Divide Fractions by Integers (2)

Notes and Guidance

Children divide fractions where the numerator is not a multiple of the integer they are dividing by.

They draw diagrams to divide fractions into equal parts and explore the link between multiplying by a unit fraction and dividing by an integer.

Children find equivalent fractions to support the divisions and draw diagrams to model how this works.

Mathematical Talk

How is Mo's method of dividing fractions similar to multiplying $\frac{1}{3}$ by $\frac{1}{2}$?

Do you prefer Mo's or Annie's method? Explain why.

Why does finding an equivalent fraction help us to divide fractions by integers?

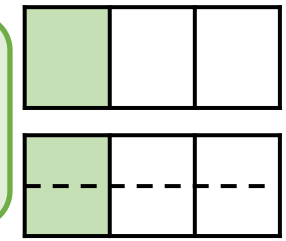
What multiplication can I use to calculate $\frac{3}{5} \div 2$? Explain how you know.

Varied Fluency

Mo is dividing $\frac{1}{3}$ by 2

I have divided one third into 2 equal parts. Each part is worth $\frac{1}{6}$

$\frac{1}{3} \div 2 = \frac{1}{6}$

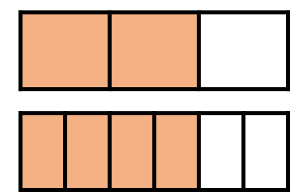


Draw diagrams to calculate:

$\frac{1}{3} \div 3 =$ $\frac{2}{3} \div 3 =$ $\frac{1}{5} \div 3 =$ $\frac{2}{5} \div 3 =$

Annie is dividing $\frac{2}{3}$ by 4

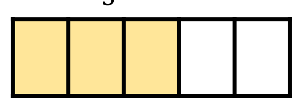
The numerator isn't a multiple of the integer I am dividing by so I will find an equivalent fraction to help me divide the numerator equally.



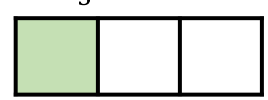
$\frac{2}{3} = \frac{4}{6}$ $\frac{4}{6} \div 4 = \frac{1}{6}$

Find equivalent fractions to calculate:

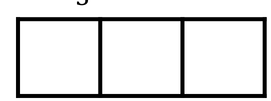
$\frac{3}{5} \div 2$



$\frac{1}{3} \div 3$



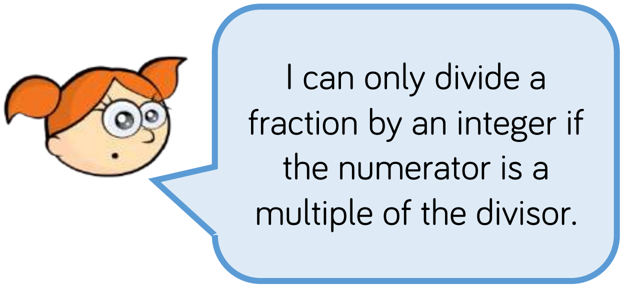
$\frac{2}{3} \div 3$



Divide Fractions by Integers (2)

Reasoning and Problem Solving

Alex says,



Do you agree?
Explain why.

Alex is wrong, we can divide any fraction by an integer.

Calculate the missing fractions and integers.

$$\square \div 4 = \frac{7}{36}$$

$$\frac{3}{20} \div \square = \frac{3}{80}$$

$$\square \div \square = \frac{2}{5}$$

Is there more than one possibility?

$\frac{7}{9}$

4

There are many possibilities in this last question. Children could look for patterns between the fractions and integers.

Four Rules with Fractions

Notes and Guidance

Children combine the four operations when calculating with fractions.
This is a good opportunity to recap the order of operations as children calculate equations with and without brackets.
Encourage children to draw bar models to represent worded problems in order to understand which operation they need to use?

Mathematical Talk

Which part of the equation do we calculate first when we have more than one operation?

What do you notice about the six questions that begin with $3\frac{1}{3}$?

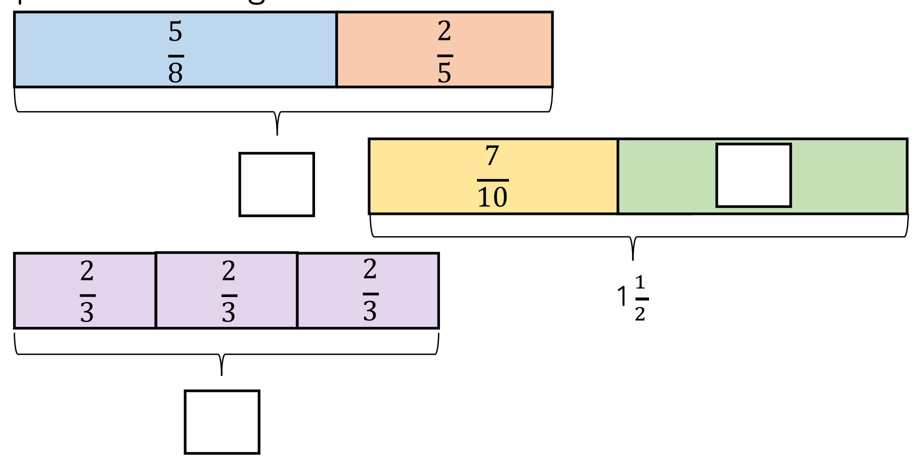
What's the same about the equations? What's different?

Which equation has the largest answer? Can you order the answers to the equations in descending order?

Can you write the worded problem as a number sentence?

Varied Fluency

- Complete the missing boxes.



- Calculate:

$3\frac{1}{3} + \frac{1}{3} - 2 =$ $\quad$ $3\frac{1}{3} + \frac{1}{3} + 2 =$ $\quad$ $3\frac{1}{3} + \frac{1}{3} \times 2 =$

$3\frac{1}{3} + \frac{1}{3} \div 2 =$ $\quad$ $(3\frac{1}{3} + \frac{1}{3}) \times 2 =$ $\quad$ $(3\frac{1}{3} + \frac{1}{3}) \div 2 =$

- Jack has one quarter of a bag of sweets and Whitney has two thirds of a bag of sweets. They combined their sweets and shared them equally between themselves and Rosie.
What fraction of the sweets does each child receive?

Four Rules with Fractions

Reasoning and Problem Solving

Add two sets of brackets to make the following calculation correct:

$$\frac{1}{2} + \frac{1}{4} \times 8 + \frac{1}{6} \div 3 = 6\frac{1}{18}$$

Explain where the brackets go and why. Did you find any difficulties?

$(\frac{1}{2} + \frac{1}{4}) \times 8 + (\frac{1}{6} \div 3)$

Match each calculation to the correct answer.

$(\frac{2}{3} + \frac{2}{9}) \div 4$	$\frac{5}{9}$
$\frac{2}{3} - \frac{1}{3} \div 3$	$\frac{2}{9}$
$\frac{1}{3} \times 2 - (1\frac{1}{9} \div 2)$	$\frac{1}{9}$

$\left(\frac{2}{3} + \frac{2}{9}\right) \div 4 = \frac{2}{9}$

$\frac{2}{3} - \frac{1}{3} \div 3 = \frac{5}{9}$

$\frac{1}{3} \times 2 - (1\frac{1}{9} \div 2) = \frac{1}{9}$

Fraction of an Amount

Notes and Guidance

Children calculate fractions of an amount. They recognise that the denominator is the number of parts the amount is being divided into, and the numerator is the amount of those parts we need to know about.
Encourage children to draw bar models to support the procedure of dividing by the denominator and multiplying by the numerator to find fractions of amounts.

Mathematical Talk

What is the value of the whole?

How many equal parts are there altogether?

How many equal parts do we need?

What is the value of each equal part?

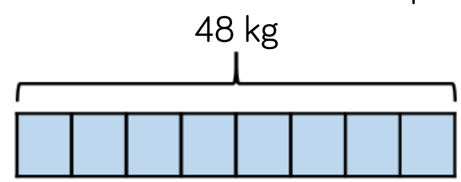
Can you see a pattern in the questions starting with $\frac{1}{5}$ of 30?

What would the next column to the right of the questions be?

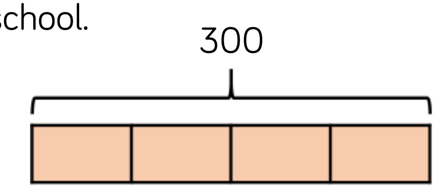
What would the next row of questions underneath be? How do you know? How can you predict the answers?

Varied Fluency

- A cook has 48 kg of potatoes. He uses $\frac{5}{8}$ of the potatoes. How many kilograms of the potatoes does he have left?
Use the bar model to find the answer to this question.



- A football team has 300 tickets to give away.
They give $\frac{3}{4}$ of them to a local school.
How many tickets are left?



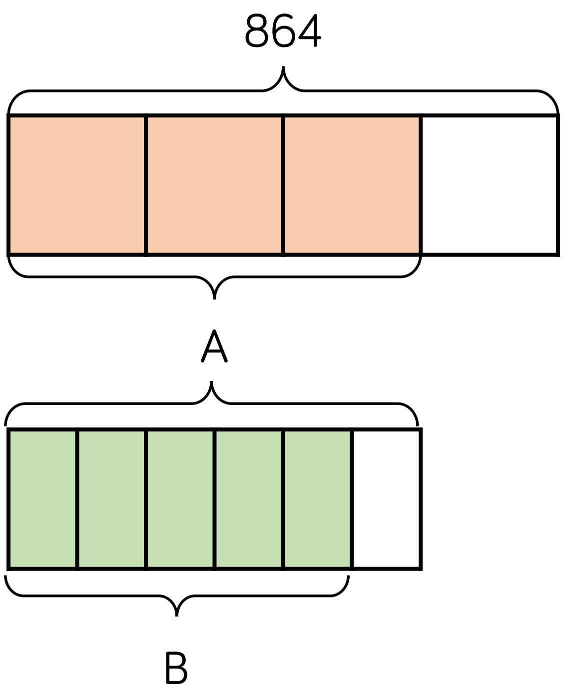
- Calculate:

$\frac{1}{5}$ of 30 =	$\frac{1}{5}$ of 60 =	$\frac{1}{5}$ of 120 =	$\frac{1}{5}$ of 240 =
$\frac{2}{5}$ of 30 =	$\frac{1}{5}$ of 600 =	$\frac{1}{10}$ of 120 =	$\frac{6}{5}$ of 240 =
$\frac{4}{5}$ of 30 =	$\frac{1}{5}$ of 6,000 =	$\frac{1}{20}$ of 120 =	$\frac{11}{5}$ of 240 =

Fraction of an Amount

Reasoning and Problem Solving

What is the value of A?
What is the value of B?



A = 648
B = 540

Two fashion designers receive $\frac{3}{8}$ of 208 metres of material.

One of them says:

Is she correct?
Explain your reasoning.

She is incorrect because 26 is only one eighth of 208
She needs to multiply her answer by 3 so that they each get 78 m each.

Calculate the missing digits.

$\frac{3}{8}$ of 40 = $\frac{?}{10}$ of 150

$\frac{1}{5}$ of 315 = $\frac{?}{8}$ of 72

1

7

Find the Whole

Notes and Guidance

Children find the whole amount from the known value of a fraction. Encourage children to continue to use bar models to support them in representing the parts and the whole.
Children will consider looking for patterns when calculating the whole. Highlight the importance of multiplication and division when calculating fractions of amounts and how knowing our times-tables can support us to calculate the whole more efficiently.

Mathematical Talk

How many equal parts are there altogether?

How many equal parts do we know?

What is the value of each equal part?

What is the value of the whole?

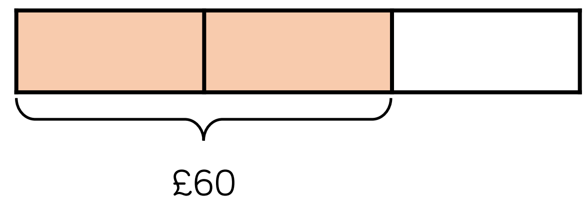
Can you see a pattern in the questions ?

How can we find the whole?

Can you estimate what the answer is? Can you check the answer using a bar model?

Varied Fluency

Jack has spent $\frac{2}{3}$ of his money.
He spent £60, how much did he have to start with?



Use a bar model to represent and solve the problems.

- Rosie eats $\frac{2}{5}$ of a packet of biscuits. She eats 10 biscuits. How many biscuits were in the original packet?
- In an election, $\frac{3}{8}$ of a town voted. If 120 people voted, how many people lived in the town?

Calculate:

$\frac{1}{4}$ of ____ = 12 $\quad$ $\frac{1}{4}$ of ____ = 36 $\quad$ $\frac{1}{4}$ of ____ = 108

$\frac{1}{12}$ of ____ = 12 $\quad$ $\frac{3}{4}$ of ____ = 36 $\quad$ $\frac{4}{4}$ of ____ = 108

Find the Whole

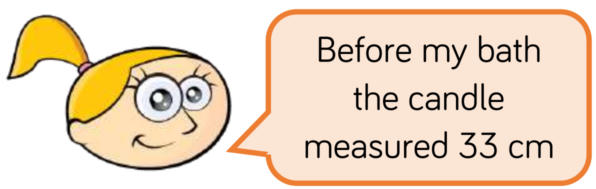
Reasoning and Problem Solving

Eva lit a candle while she had a bath.

After her bath, $\frac{2}{5}$ of the candle was left.

It measured 13 cm.

Eva says:



Is she correct?

Explain your reasoning.

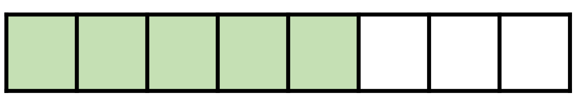
She is incorrect.

$13 \div 2 = 6.5$

$6.5 \times 5 = 32.5$cm

She either didn't halve correctly or didn't multiply correctly

Write a problem which this bar model could represent.

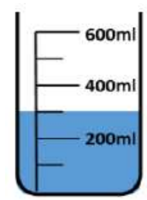


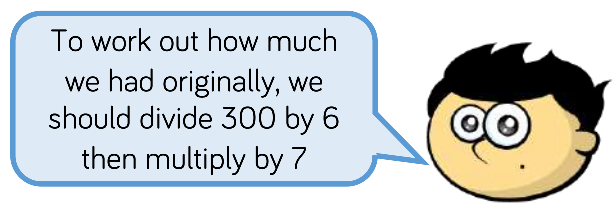
Many possibilities.

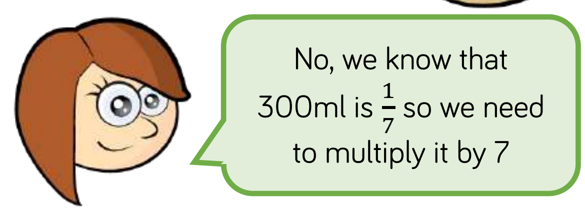
$\frac{5}{8}$ of children have blue eyes. 15 children do not have blue eyes. How many children are there altogether?

Rosie and Jack are making juice.

They use $\frac{6}{7}$ of the water in a jug and are left with this amount of water:







Who is correct?

Explain your reasoning.

Rosie is correct. Jack would only be correct if $\frac{6}{7}$ was **remaining** but $\frac{6}{7}$ is what was used. Rosie recognised that $\frac{1}{7}$ is left in the jug therefore multiplied it by 7 to correctly find the whole.

White Rose Maths

Autumn - Block 4

Position and Direction

Overview

Small Steps

- The first quadrant
- Four quadrants
- Translations
- Reflections

Notes for 2020/21

Position and direction was probably missed in the summer of Y5 so treat this topic as brand new learning.

The First Quadrant

Notes and Guidance

Children recap work from Year 4 and Year 5 by reading and plotting coordinates in the first quadrant (the quadrant where both x and y coordinates are positive.).

Children draw shapes on a 2-D grid from given coordinates and may use their increasing understanding to write coordinates for shapes without plotting the points.

Mathematical Talk

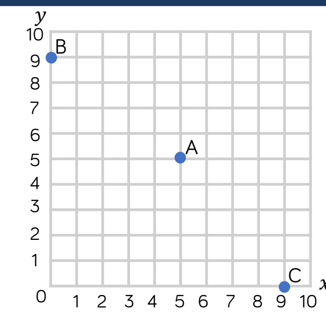
Which axis do we look at first?

Does joining up the vertices already given help you to draw the shape?

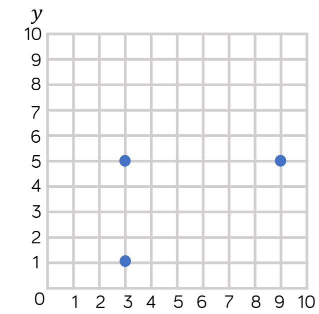
Can you draw a shape in the first quadrant and describe the coordinates of the vertices to a friend?

Varied Fluency

Whitney plots three coordinates.
Write down the coordinates of points A, B and C.



Tommy is drawing a rectangle on a grid.
Plot the final vertex of the rectangle.
Write the coordinate of the final vertex.

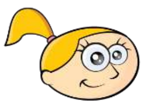


Draw the vertices of the polygon with the coordinates (7, 1), (7, 4) and (10, 1)
What type of polygon is the shape?

The First Quadrant

Reasoning and Problem Solving

Eva is drawing a trapezium.
She wants her final shape to look like this:



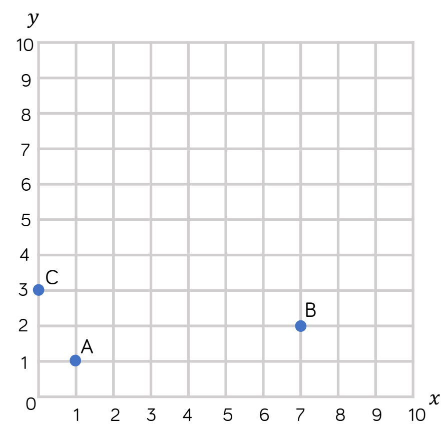
Eva uses the coordinates (2, 4), (4, 5), (1, 6) and (5, 6).
Will she draw the shape that she wants to?
If not, can you correct her coordinates?

Eva has plotted the coordinate (4, 5) incorrectly. This should be plotted at (4, 4) to make the trapezium that she wanted to draw (an isosceles trapezium).

Mo has written the coordinates of points A, B and C.

A (1, 1) **B** (2, 7) **C** (3, 0)

Mark Mo's work and correct his mistakes.



Explain why Mo could not make the same mistake for point A as he made for points B and C.

A is correct.

B and C have been plotted incorrectly because Mo has plotted the x and y coordinates the wrong way round.

Because the coordinates for point A are both the same number it does not matter if Mo incorrectly reads the y coordinate as the first and the x coordinate as the second.

Four Quadrants

Notes and Guidance

Children extend their knowledge of the first quadrant to read and plot coordinates in all four quadrants.
They draw shapes from coordinates given.
Children need to become fluent in deciding which part of the axis is positive or negative.
Children need to develop understanding of how to find the length of a line by using the coordinates of its two endpoints.

Mathematical Talk

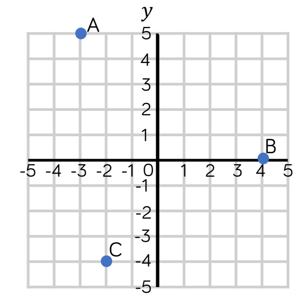
Which axis do we look at first?

If (0, 0) is the centre of the axis (the origin), which way do you move along the x-axis to find negative coordinates?

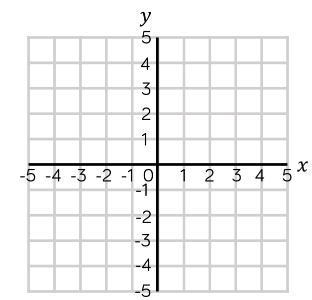
Which way do you move along the y-axis to find negative coordinates?

Varied Fluency

- Dora plotted three coordinates.
  Write down the coordinates of points A, B and C.

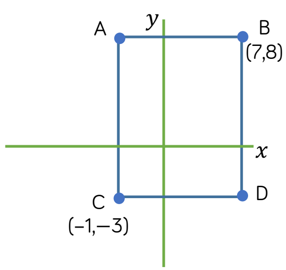


- Draw a shape using the coordinates (-2, 2), (-4, 2), (-2, -3) and (-4, -2).
  What is the name of shape?



- Work out the missing coordinates of the rectangle.

  What is the length of side AB?



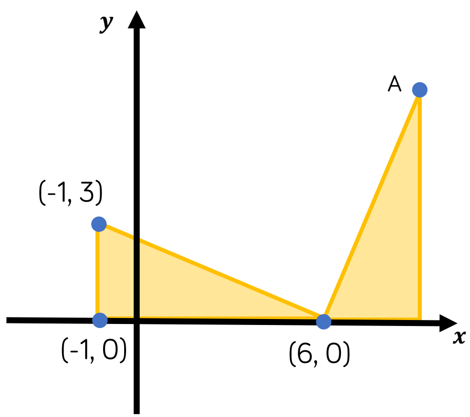
Four Quadrants

Reasoning and Problem Solving

The diagram shows two identical triangles.

The coordinates of three points are shown.

Find the coordinates of point A.



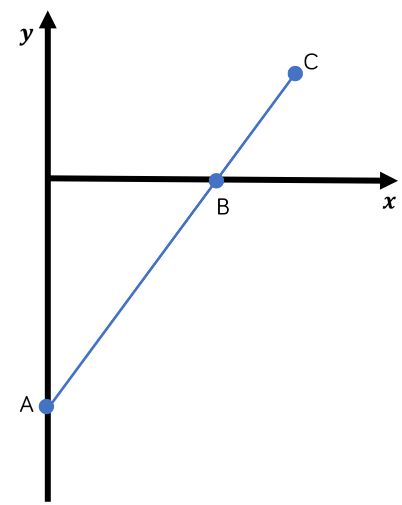
(9, 7)

A is the point (0, – 10)

B is the point (8, 0)

The distance from A to B is two thirds of the distance from A to C.

Find the coordinates of C.



(12, 5)

Translations

Notes and Guidance

Children use knowledge of coordinates and positional language to translate shapes in all four quadrants.

They describe translations using directional language, and use instructions to draw translated shapes.

Mathematical Talk

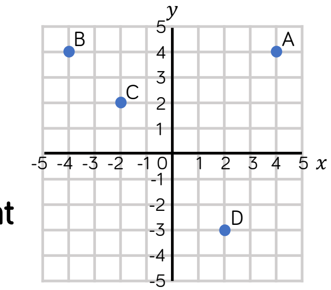
What does translation mean?

Which point are you going to look at when describing the translation?

Does each vertex translate in the same way?

Varied Fluency

Use the graph to describe the translations.
One has been done for you.



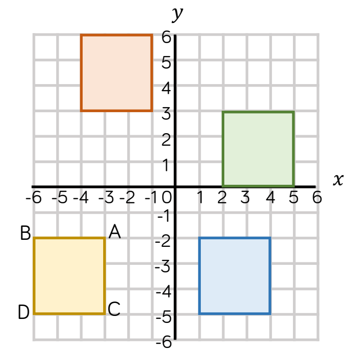
From A to B translate **8** units to the **left**.

From C to D translate __ units to the **right** and __ units **down**.

From D to B translate **6** units to the _____ and **7** units _____.

From A to C translate __ units to the ____ and __ units ____.

Write the coordinates for vertices A, B, C and D.
Describe the translation of ABCD to the blue square.



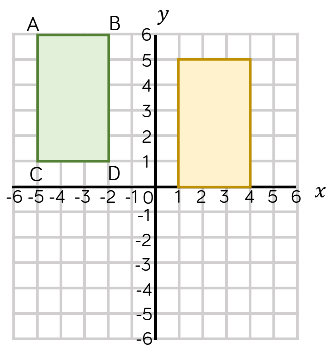
ABCD is moved 2 units to the right and 8 units up. Which colour square is it translated to?
Write the coordinates of the vertices of the translated shape.

Translations

Reasoning and Problem Solving

True or False?

Dexter has translated the rectangle ABCD 6 units down and 1 unit to the right to get to the yellow rectangle.

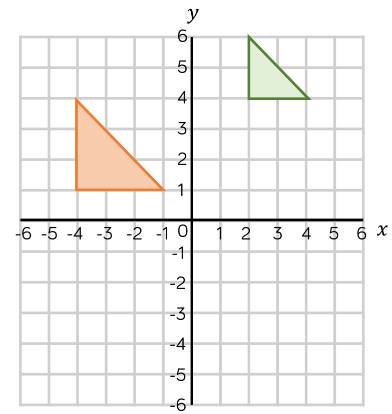


Explain your reasoning.

False.
The translation is 6 units to the right and 1 unit down.

Spot the Mistake.

The green triangle has been translated 6 units to the left and 3 units down.



The triangle has changed size.
When a shape is translated its size does not change.

-6

Reflections

Notes and Guidance

Children extend their knowledge of reflection by reflecting shapes in four quadrants. They will reflect in both the x-axis and the y-axis.

Children should use their knowledge of coordinates to ensure that shapes are correctly reflected.

Mathematical Talk

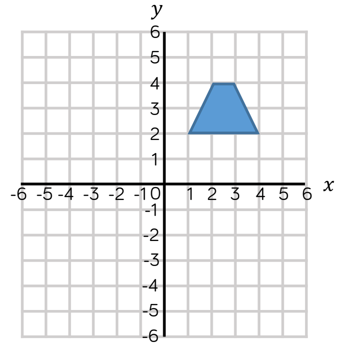
How is reflecting different to translating?

Can you reflect one vertex at a time? Does this make it easier to reflect the shape?

Which axis are you going to use as the mirror line?

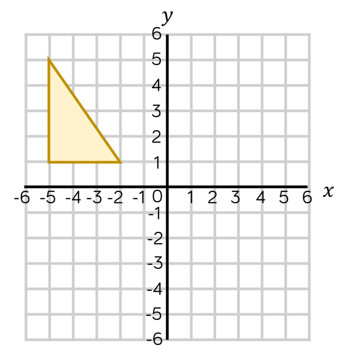
Varied Fluency

Reflect the trapezium in the x-axis and then the y –axis. Complete the table with the new coordinates of the shape.



	Reflected in the x-axis	Reflected in the y-axis
(1, 2)		
(4, 2)		
(2, 4)		
(3, 4)		

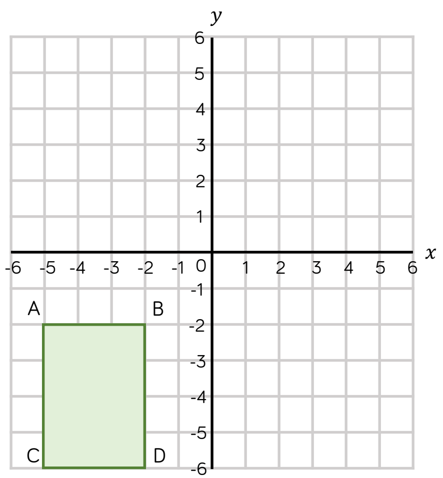
Translate the shape 4 units to the right. Then reflect the translated shape in the y-axis.



Reflections

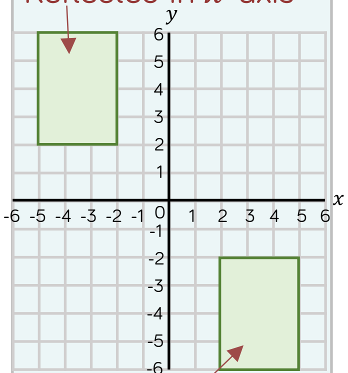
Reasoning and Problem Solving

Rectangle ABCD is the result of a rectangle being reflected in either the x-or the y-axis.
Where could the original rectangle have been? Draw the possible original rectangles on the coordinate grid, and label the coordinates of each vertex.



The two original rectangles are:

Reflected in x-axis

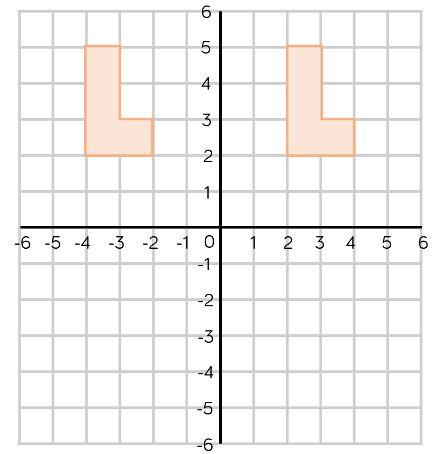


Reflected in y-axis

x-axis reflection original coordinates:
(-5, 6), (-2, 6), (-5, 2), (-2, 2)

y-axis reflection original coordinates:
(2, -2), (5, -2),
(2, -6), (5, -6)

Annie has reflected the shape in the y-axis.
Is her drawing correct?
If not explain why.



Annie has used the correct axis, but her shape has not been reflected. She has just drawn the shape again on the other side of the axis.

Spring Scheme of Learning

Year 6

#MathsEveryoneCan

2020-21

White Rose Maths

New for 2020/21

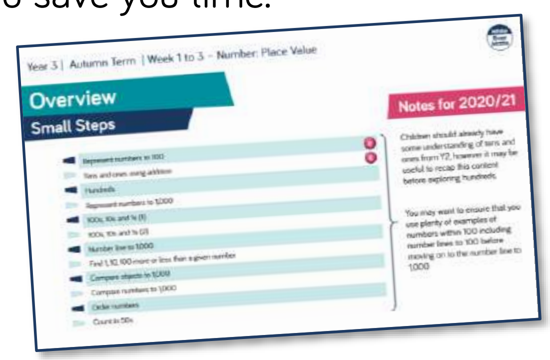
2020 will go down in history. The world has changed for all of us.

We want to do as much as we can to support children, teachers, parents and carers in these very uncertain times.

We have amended our schemes for 2020/21 to:

- ★ highlight key teaching points
- ★ recap essential content that children may have forgotten
- ★ flag any content that you might not have covered during the school closures period.

We hope these changes will add further value to the schemes and save you time.



Lesson-by-lesson overviews

We've always been reluctant to produce lesson-by-lesson overviews as every class is individual and has different needs. However, many of you have said that if blended learning becomes a key feature of school life next year, a weekly plan with linked content and videos could be really useful.

As always, we've listened! We've now produced a complete lesson-by-lesson overview for Y1 to Y9 that schools can use or adapt as they choose. Each lesson will be linked to a free-to-use home learning video, and for premium subscribers, a worksheet. This means that you can easily assign work to your class, whether they are working at home or in school.

Inevitably, this lesson-by-lesson structure won't suit everyone, but if it works for you, then please do make use of this resource as much as you wish.

Teaching for Mastery

These overviews are designed to support a mastery approach to teaching and learning and have been designed to support the aims and objectives of the new National Curriculum.

The overviews:

- have number at their heart. A large proportion of time is spent reinforcing number to build competency
- ensure teachers stay in the required key stage and support the ideal of depth before breadth.
- ensure students have the opportunity to stay together as they work through the schemes as a whole group
- provide plenty of opportunities to build reasoning and problem solving elements into the curriculum.

For more guidance on teaching for mastery, visit the NCETM website:

https://www.ncetm.org.uk/resources/47230

Concrete - Pictorial - Abstract

We believe that all children, when introduced to a new concept, should have the opportunity to build competency by taking this approach.

Concrete – children should have the opportunity to use concrete objects and manipulatives to help them understand what they are doing.

Pictorial – alongside this children should use pictorial representations. These representations can then be used to help reason and solve problems.

Abstract – both concrete and pictorial representations should support children's understanding of abstract methods.

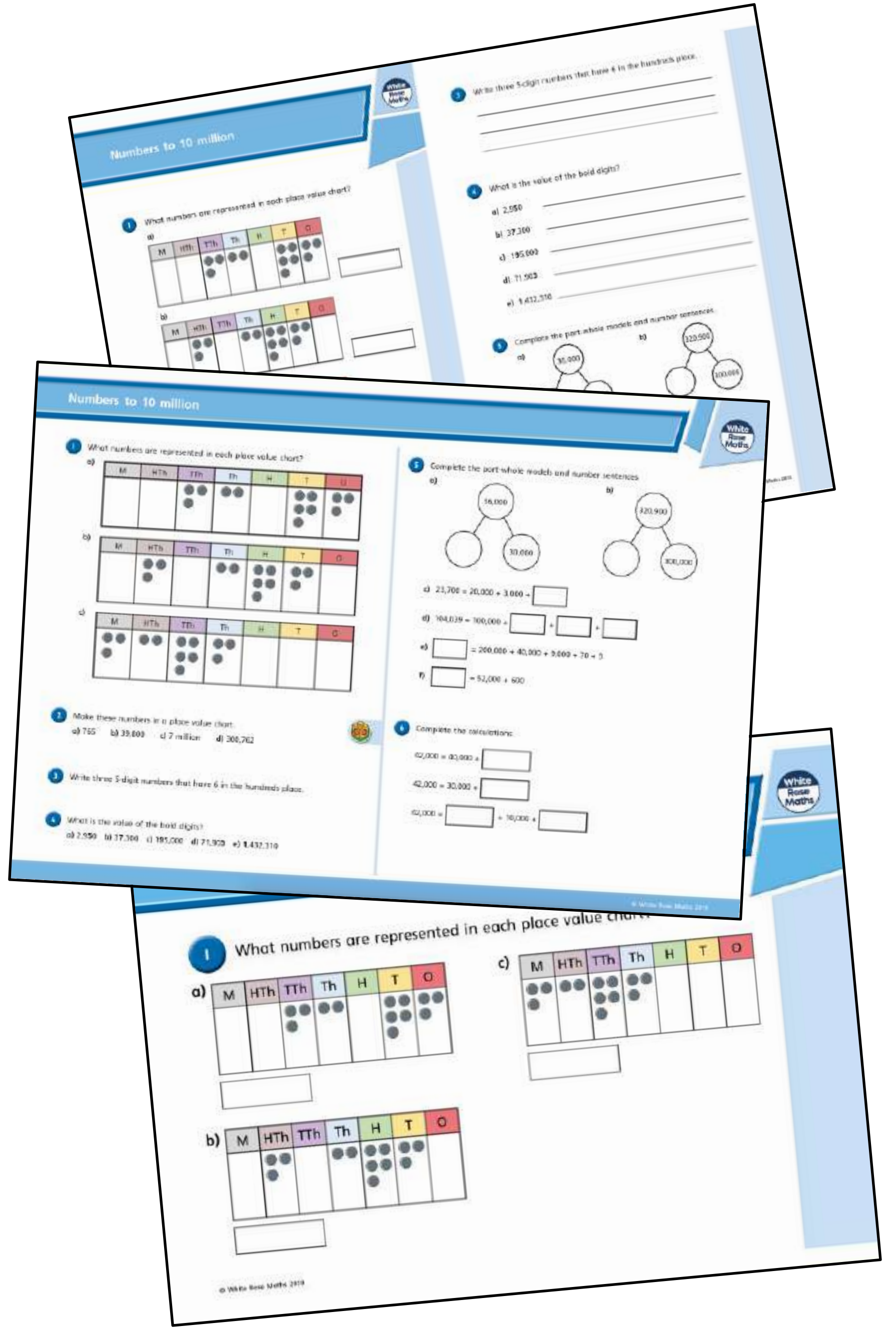
Need some CPD to develop this approach? Visit www.whiterosemaths.com for find a course right for you.

Supporting resources

We have produced supporting resources for every small step from Year 1 to Year 11.

The worksheets are provided in three different formats:

- Write on worksheet – ideal for children to use the ready made models, images and stem sentences.
- Display version – great for schools who want to cut down on photocopying.
- PowerPoint version – one question per slide. Perfect for whole class teaching or mixing questions to make your own bespoke lesson.



For more information visit our online training and resources centre resources.whiterosemaths.com or email us directly at support@whiterosemaths.com

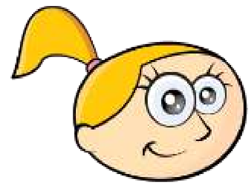
Meet the Characters

Children love to learn with characters and our team within the scheme will be sure to get them talking and reasoning about mathematical concepts and ideas. Who's your favourite?

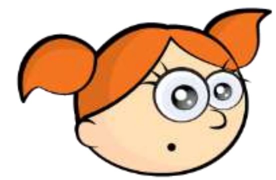
Teddy

Rosie

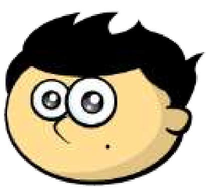
Mo



Eva

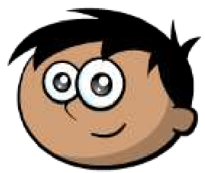


Alex

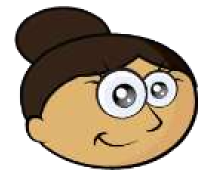


Jack

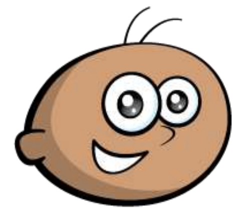
Whitney



Amir



Dora



Tommy

Dexter

Ron

Annie

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	Number: Place Value		Number: Addition, Subtraction, Multiplication and Division					Number: Fractions				Geometry: Position and Direction
Spring	Number: Decimals		Number: Percentages		Number: Algebra		Measurement: Converting Units	Measurement: Perimeter, Area and Volume		Number: Ratio		Statistics
Summer	Geometry: Properties of Shape			Consolidation or SATs preparation		Consolidation, investigations and preparations for KS3						

White Rose Maths

Spring - Block 1

Decimals

Overview

Small Steps

- Decimals up to 2 decimal places (R)
- Understand thousandths (R)
- Three decimal places
- Multiply by 10, 100 and 1,000
- Divide by 10, 100 and 1,000
- Multiply decimals by integers
- Divide decimals by integers
- Division to solve problems
- Decimals as fractions
- Fractions to decimals (1)
- Fractions to decimals (2)

Notes for 2020/21

The recap steps are at the beginning of this block to ensure children have a good understanding of numbers up to three decimal places before moving on to multiplication and division.

This should build on place value work in the autumn term and make use of place value grids and counters to build on previous learning.

Decimals up to 2 d.p.

Notes and Guidance

Children use place value counters and a place value grid to make numbers with up to two decimal places.

They read and write decimal numbers and understand the value of each digit.

They show their understanding of place value by partitioning decimal numbers in different ways.

Mathematical Talk

How many ones/tenths/hundredths are in the number?
How do we write this as a decimal? Why?

What is the value of the ____ in the number _____?

When do we need to use zero as a place holder?

How can we partition decimal numbers in different ways?

Varied Fluency

R

Which number is represented on the place value chart?

Ones	Tenths	Hundredths
	0.1	0.01 0.01
0	1	2

There are ____ ones, ____ tenths and ____ hundredths.

The number is ____

Represent the numbers on a place value chart and complete the stem sentences.

0.28 | 0.65 | 0.07 | 1.26

Make the numbers with place value counters and write down the value of the underlined digit.

2.<u>4</u>5 | 3.0<u>4</u> | <u>4</u>.44 | <u>4</u>3.34

0.76 = 0.7 + 0.06 = 7 tenths and 6 hundredths.
Fill in the missing numbers.

0.83 = ____ + 0.03 = ____________ and 3 hundredths.

0.83 = 0.7 + ____ = 7 tenths and ____________

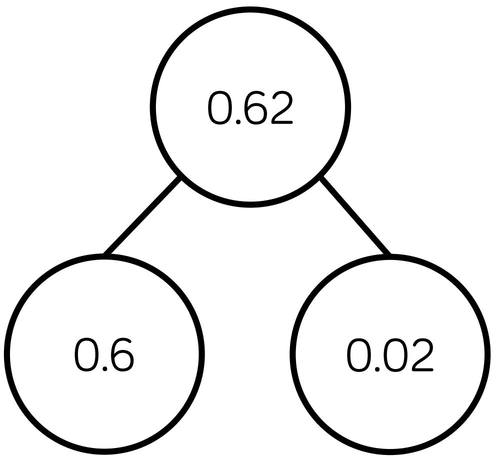
How many other ways can you partition 0.83?

Decimals up to 2 d.p.

Reasoning and Problem Solving

R

Dexter says there is only one way to partition 0.62



Prove Dexter is incorrect by finding at least three different ways of partitioning 0.62

0.62 = 0.12 + 0.5

0.62 = 0.4 + 0.22

0.62 = 0.3 + 0.32

0.62 = 0.42 + 0.2

0.62 = 0.1 + 0.52

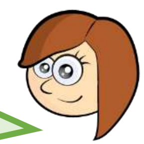
0.62 = 0.03 + 0.59

etc.

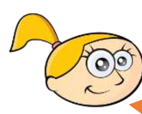
Match each description to the correct number.

Teddy: My number has the same amount of tens and tenths.

Amir: My number has one decimal place.



Rosie: My number has two hundredths.



Eva: My number has six tenths.

46.2 | 2.64 | 46.02 | 40.46

Teddy – 40.46
Amir – 46.2
Rosie – 46.02
Eva – 2.64

Understand Thousandths

Notes and Guidance

Children build on previous learning of tenths and hundredths and apply this to understanding thousandths.
Opportunities to develop understanding of thousandths through the use of concrete and pictorial representations need to be incorporated.
When exploring the relationships between tenths, hundredths and thousandths, consider decimal and mixed number equivalences.

Mathematical Talk

If 4 tenths = 0.4, 4 hundredths = 0.04, what is 4 thousandths equal to?

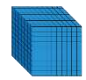
Using the place value charts:

- How many tenths are in a whole?
- How many hundredths are there in 1 tenth?
- Using place value counters complete the final chart.
- How many thousandths in 1 hundredth?

Varied Fluency

R

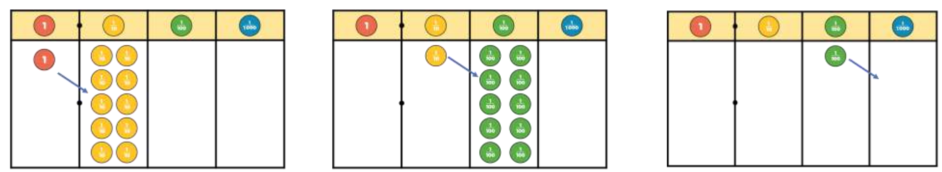
Eva is using Base 10 to represent decimals.

 = 1 whole  = 1 tenth = 1 hundredth  = 1 thousandth

Use Base 10 to build:

- 4 wholes, 4 tenths, 4 hundredths, 4 thousandths
- 5 tenths, 7 hundredths and 5 thousandths
- 2.357

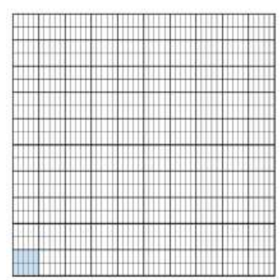
Use the place value counters to help you fill in the final chart.



= ___ tenths 

= ___hundredths 

= ___ thousandths



What has this hundred square been divided up into?
How many thousandths are there in one hundredth?
How many thousandths are in one tenth?

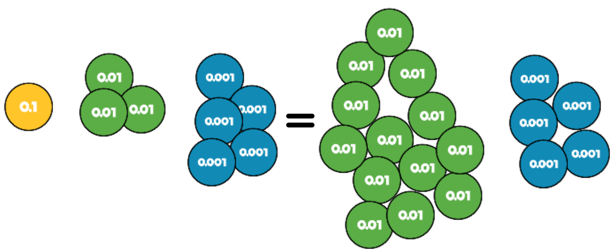
White Rose Maths

Understand Thousandths

Reasoning and Problem Solving

R

Rosie thinks the 2 values are equal.



Do you agree?
Explain your thinking.

Can you write this amount as a decimal and as a fraction?

Agree.

We can exchange ten hundredth counters for one tenth counter.

$0.135 = \frac{135}{1000}$

0.394

$= 3$ tenths, 9 hundredths and 4 thousandths

$= \frac{3}{10} + \frac{9}{100} + \frac{4}{1000}$

$= 0.3 + 0.09 + 0.004$

Write these numbers in three different ways:

0.472 0.529 0.307

$0.472 = 4$ tenths, seven hundredths and 2 thousandths $= \frac{4}{10} + \frac{7}{100} + \frac{2}{1000} = 0.4 + 0.07 + 0.002$

$0.529 = 5$ tenths, two hundredths and 9 thousandths $= \frac{5}{10} + \frac{2}{100} + \frac{9}{1000} = 0.5 + 0.02 + 0.009$

$0.307 = 3$ tenths and 7 thousandths $= \frac{3}{10} + \frac{7}{1000} = 0.3 + 0.007$

Three Decimal Places

Notes and Guidance

Children recap their understanding of numbers with up to 3 decimal places. They look at the value of each place value column and describe its value in words and digits.

Children use concrete resources to investigate exchanging between columns e.g. 3 tenths is the same as 30 hundredths.

Mathematical Talk

How many tenths are there in the number? How many hundredths? How many thousandths?

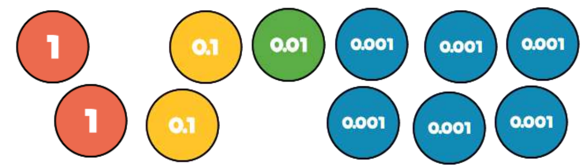
Can you make the number on the place value chart?

How many hundredths are the same as 5 tenths?

What is the value of the zero in this number?

Varied Fluency

- Complete the sentences.



There are ____ ones, ____ tenths, ____ hundredths and ____ thousandths.
The number in digits is ____________

- Use counters and a place value chart to represent these numbers.

3.456   72.204   831.07

Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths

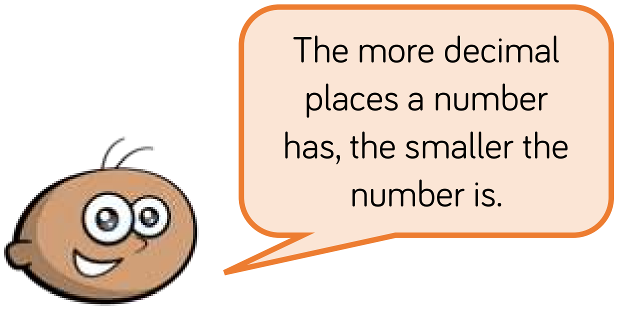
- Write down the value of the 3 in the following numbers.

0.53   362.44   739.8   0.013   3,420.98

Three Decimal Places

Reasoning and Problem Solving

Tommy says,



Do you agree?
Explain why.

Possible answer:

I do not agree with this as the number 4.39 is smaller than the number 4.465, which has more decimal places.

Alex says that 3.24 can be written as 2 ones, 13 tenths and 4 hundredths.

Do you agree?

How can you partition 3.24 starting with 2 ones?
How can you partition 3.24 starting with 1 one?
Think about exchanging between columns.

Possible answer:

I disagree; Alex's numbers would total 3.34. I could make 3.24 by having 2 ones, 12 tenths and 4 hundredths or 1 one, 22 tenths and 4 hundredths.

Four children are thinking of four different numbers.

3.454	4.445
4.345	3.54

Teddy: "My number has four hundredths."

Alex: "My number has the same amount of ones, tenths and hundredths."

Dora: "My number has less ones that tenths and hundredths."

Jack: "My number has 2 decimal places."

Match each number to the correct child.

Teddy: 4.345

Alex: 4.445

Dora: 3.454

Jack: 3.54

Multiply by 10, 100 and 1,000

Notes and Guidance

Children multiply numbers with up to three decimal places by 10, 100 and 1,000
They discover that digits move to the left when they are multiplying and use zero as a place value holder. The decimal point does not move.
Once children are confident in multiplying by 10, 100 and 1,000, they use these skills to investigate multiplying by multiples of these numbers e.g. 2.4 × 20

Mathematical Talk

What number is represented on the place value chart?

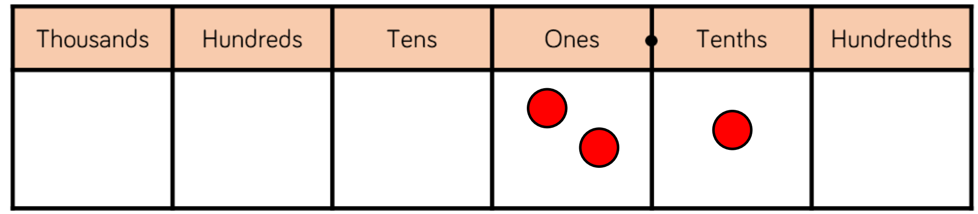
Why is 0 important when multiplying by 10, 100 and 1,000?

What patterns do you notice?

What is the same and what is different when multiplying by 10, 100, 1,000 on the place value chart compared with the Gattegno chart?

Varied Fluency

Identify the number represented on the place value chart.



Multiply it by 10, 100 and 1,000 and complete the sentence stem for each.
When multiplied by ____ the counters move ____ places to the ______.

Use a place value chart to multiply the following decimals by 10, 100 and 1,000

6.4 | 6.04 | 6.004

Fill in the missing numbers in these calculations

32.4 × ☐ = 324 | 1.562 × 1,000 = ☐

☐ × 100 = 208 | 4.3 × ☐ = 86

Multiply by 10, 100 and 1,000

Reasoning and Problem Solving

Using the digit cards 0-9 create a number with up to 3 decimal places e.g. 3.451
Cover the number using counters on your Gattegno chart.

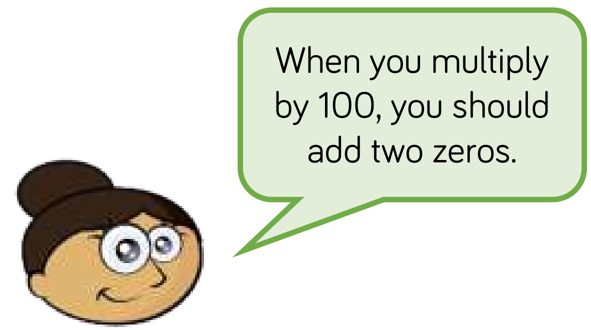
10,000	20,000	30,000	40,000	50,000	60,000	70,000	80,000	90,000
1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009

Explore what happens when you multiply your number by 10, then 100, then 1,000

What patterns do you notice?

Children will be able to see how the counter will move up a row for multiplying by 10, two rows for 100 and three rows for 1,000. They can see that this happens to each digit regardless of the value.
For example, 3.451×10 becomes 34.51
Each counter moves up a row but stays in the same column.

Dora says,



Do you agree?
Explain your thinking.

Children should explain that when you multiply by 100 the digits move two places to the left.

For example:
$0.34 \times 100 = 0.3400$ is incorrect as 0.34 is the same as 0.3400
Also:
$0.34 + 0 + 0 = 0.34$

Children show
$0.34 \times 100 = 34$

White Rose Maths

Divide by 10, 100 and 1,000

Notes and Guidance

Once children understand how to multiply decimals by 10, 100 and 1,000, they can apply this knowledge to division, which leads to converting between units of measure.

It is important that children continue to understand the importance of 0 as a place holder. Children also need to be aware that 2.4 and 2.40 are the same. Similarly, 12 and 12.0 are equivalent.

Mathematical Talk

What happens to the counters/digits when you divide by 10, 100 or 1,000?

Why is zero important when dividing by 10, 100 and 1,000?

What is happening to the value of the digit each time it moves one column to the right?

What are the relationships between tenths, hundredths and thousandths?

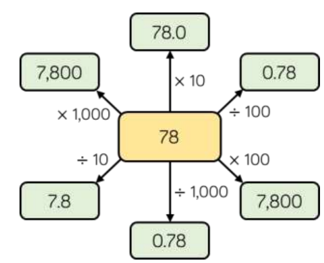
Varied Fluency

- Use the place value chart to divide the following numbers by 10, 100 and 1,000

Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths

44 1.36 107 5

- Tick the correct answers. Can you explain the mistakes with the incorrect answers?



- Complete the table.

	÷10	÷ 100	÷ 1,000
30			
3 kg			
	0.9		
			9.0
		9.09	

Divide by 10, 100 and 1,000

Reasoning and Problem Solving

Using the following rules, how many ways can you make 70?

- Use a number from column A
- Use an operation from column B.
- Use number from column C.

A	B		C
0.7	×	÷	0.1
7			1
70			10
700			100
7,000			1,000

Possible answers:

0.7 × 100
7 × 10
70 × 1
700 ÷ 10
7,000 ÷ 100
70 ÷ 1

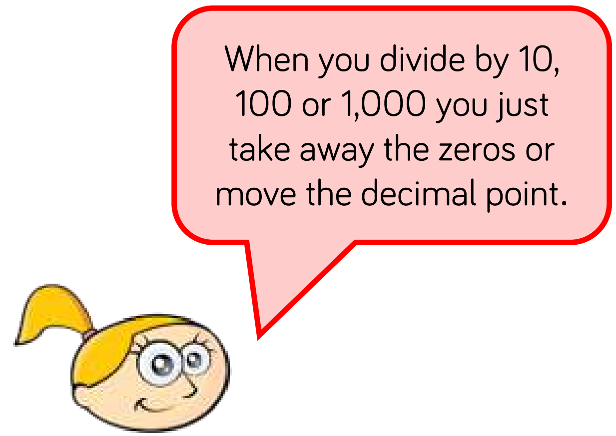
Can you find a path from 6 to 0.06?
You cannot make diagonal moves.

6	× 10	× 10	÷ 100
÷ 10	× 100	× 100	÷ 10
× 10	÷ 10	÷ 1,000	÷ 100
÷ 1,000	× 1,000	× 100	0.06

Is there more than one way?

6	× 10	× 10	÷ 100
÷ 10	× 100	× 100	÷ 10
× 10	÷ 10	÷ 1,000	÷ 100
÷ 1,000	× 1,000	× 100	0.06

Eva says,



Do you agree?
Explain why.

Eva is wrong, the decimal point never moves. When dividing, the digits move right along the place value columns.

Possible examples to prove Eva wrong:

24 ÷ 10 = 2.4

107 ÷ 10 = 17
This shows that you cannot just remove a zero from the number

Multiply Decimals by Integers

Notes and Guidance

Children use concrete resources to multiply decimals and explore what happens when you exchange with decimals.

Children use their skills in context and make links to money and measures.

Mathematical Talk

Which is bigger, 0.1, 0.01 or 0.001? Why?

How many 0.1s do you need to exchange for a whole one?

Can you draw a bar model to represent the problem?

Can you think of another way to multiply by 5? (e.g. multiply by 10 and divide by 2).

Varied Fluency

- Use the place value counters to multiply 1.212 by 3
  Complete the calculation alongside the concrete representation.

Tens	Ones	Tenths	Hundredths	Thousandths
	1	0.1 0.1	0.01	0.001 0.001
	1	0.1 0.1	0.01	0.001 0.001
	1	0.1 0.1	0.01	0.001 0.001

- A jar of sweets weighs 1.213 kg.
  How much would 4 jars weigh?

- Rosie is saving her pocket money. Her mum says,

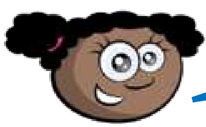
  "Whatever you save, I will give you five times the amount."

  If Rosie saves £2.23, how much will her mum give her?
  If Rosie saves £7.76, how much will her mum give her? How much will she have altogether?

Multiply Decimals by Integers

Reasoning and Problem Solving

Whitney says,



Do you agree?
Explain why.

Possible answer:

I do not agree because there are examples such as 2.23 × 2 that gives an answer with only two decimal places.

Fill in the blanks

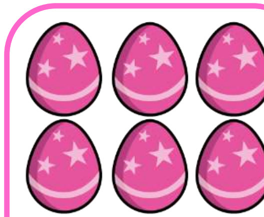
		3	•	4	5
×					
		0	•	3	0
			•	4	0
	1		•	0	0
			•		

Answer:

		3	•	4	5
×					6
		0	•	3	0
		2	•	4	0
	1	8	•	0	0
	2	0	•	7	0

Chocolate eggs can be bought in packs of 1, 6 or 8
What is the cheapest way for Dexter to buy 25 chocolate eggs?


1 chocolate egg
52p


6 chocolate eggs
£2.85


8 chocolate eggs
£4

£11.92

He should buy four packs of 6 plus an individual egg.

Divide Decimals by Integers

Notes and Guidance

Children continue to use concrete resources to divide decimals and explore what happens when exchanges take place.

Children build on their prior knowledge of sharing and grouping when dividing and apply this skill in context.

Mathematical Talk

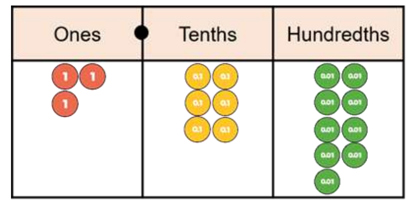
Are we grouping or sharing?

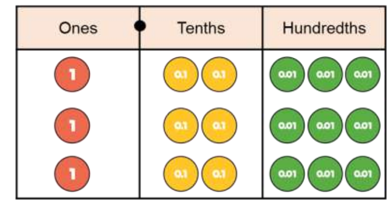
How else could we partition the number 3.69? (For example, 2 ones, 16 tenths and 9 hundredths.)

How could we check that our answer is correct?

Varied Fluency

- Divide 3.69 by 3
  Use the diagrams to show the difference between grouping and by sharing?



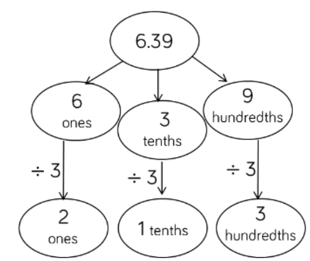


  Use these methods to complete the sentences.
  3 ones divided by 3 is _________ ones.
  6 tenths divided by 3 is __________ tenths.
  9 hundredths divided by 3 is ________ hundredths.
  Therefore, 3.69 divided by 3 is _______________

- Decide whether you will use grouping or sharing and use the place value chart and counters to solve:

  7.55 ÷ 5 8.16 ÷ 3 3.3 ÷ 6

- Amir solves 6.39 ÷ 3 using a part whole method.



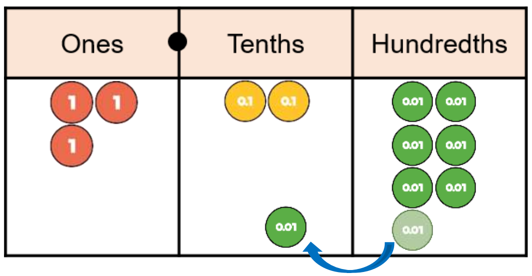
  Use this method to solve

  8.48 ÷ 2 6.9 ÷ 3 6.12 ÷ 3

Divide Decimals by Integers

Reasoning and Problem Solving

When using the counters to answer 3.27 divided by 3, this is what Tommy did:



Tommy says,

I only had 2 counters in the tenths column, so I moved one of the hundredths so each column could be grouped in 3s.

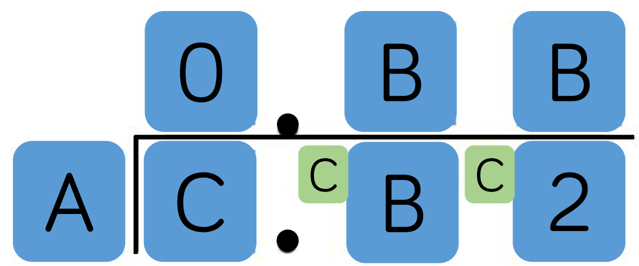
Do you agree with what Tommy has done? Explain why.

Possible answer:

Tommy is incorrect because he cannot move a hundredth to the tenths.
He should have exchanged the 2 tenths for hundredths to get an answer of 1.09

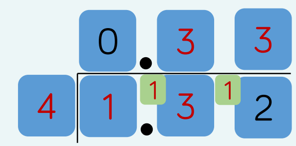
C is $\frac{1}{4}$ of A
$B = C + 2$

Use the clues to complete the division.



Children may try A as 8 and C as 2 but will realise that this cannot complete the whole division.

Therefore A is 4, B is 3 and C is 1



Division to Solve Problems

Notes and Guidance

Children will apply their understanding of division to solve problems in cases where the answer has up to 2 decimal places.

Children will continue to show division using place value counters and exchanging where needed.

Mathematical Talk

How can we represent this problem using a bar model?

How will we calculate what this item costs?

How will we use division to solve this?

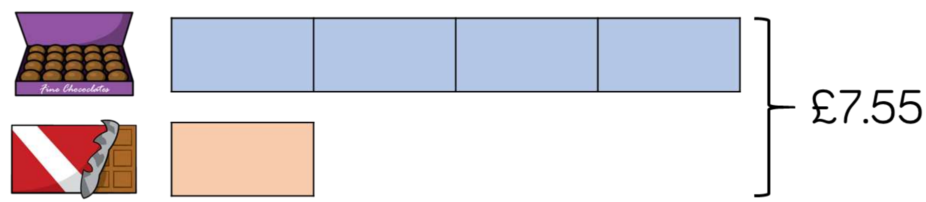
How will we label our bar model to represent this?

Varied Fluency

- Mrs Forbes has saved £4,960
  She shares the money between her 15 grandchildren.
  How much do they each receive?

- Modelling clay is sold in two different shops.
  Shop A sells four pots of clay for £7.68
  Shop B sells three pots of clay for £5.79
  Which shop has the better deal?
  Explain your answer.

- A box of chocolates costs 4 times as much as a chocolate bar.
  Together they cost £7.55



How much does each item cost?
How much more does the box of chocolates cost?

Division to Solve Problems

Reasoning and Problem Solving

Each division sentence can be completed using the digits below.

1 2 3 4 5 6

☐ . 3 ÷ ☐ = 0.26

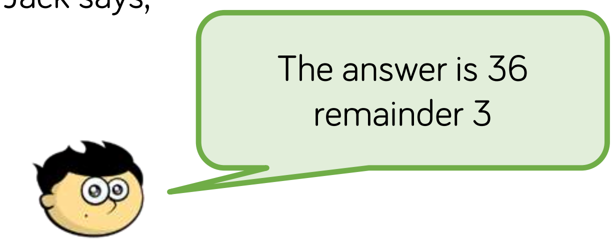
12 . ☐ ÷ ☐ = 4.2

4 . ☐ 8 ÷ ☐ = 1.07

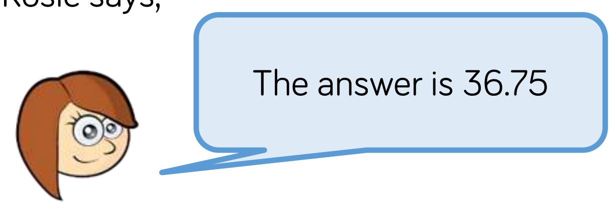
1.3 ÷ 5 = 0.26
12.6 ÷ 3 = 4.2
4.28 ÷ 4 = 1.07

Jack and Rosie are both calculating the answer to 147 ÷ 4

Jack says,



Rosie says,



Who do you agree with?

They are both correct.

Rosie has divided her remainder of 3 by 4 to get 0.75 whereas Jack has recorded his as a remainder.

Decimals as Fractions

Notes and Guidance

Children explore the relationship between decimals and fractions. They start with a decimal and use their place value knowledge to help them convert it into a fraction.
Children will use their previous knowledge of exchanging between columns, for example, 3 tenths is the same as 30 hundredths.
Once children convert from a decimal to a fraction, they simplify the fraction to help to show patterns.

Mathematical Talk

How would you record your answer as a decimal and a fraction? Can you simplify your answer?

How would you convert the tenths to hundredths?

What do you notice about the numbers that can be simplified in the table?

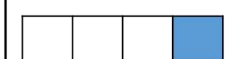
Can you have a unit fraction that is larger than 0.5? Why?

Varied Fluency

What decimal is shaded?
Can you write this as a fraction?

0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1

Complete the table.

Decimal	Fraction in tenths or hundredths	Simplified fraction
0.6	$\frac{6}{10}$	$\frac{3}{5}$
		
		
		
0.95		

Three friends share a pizza. Sam ate 0.25 of the pizza, Mark ate 0.3 of the pizza and Jill ate 0.35 of the pizza.

- Can you write the amount each child ate as a fraction?
- What fraction of the pizza is left?

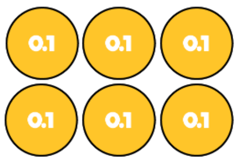
White Rose Maths

Decimals as Fractions

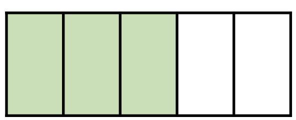
Reasoning and Problem Solving

Odd one out.

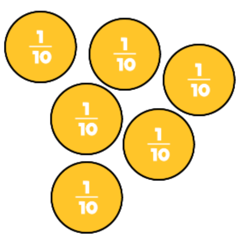
A



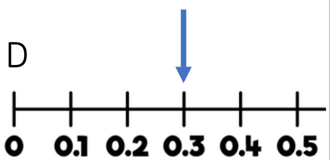
B



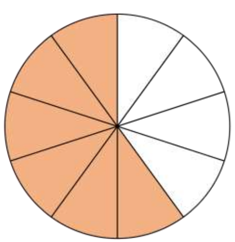
C



D



E



F

0.2 × 3

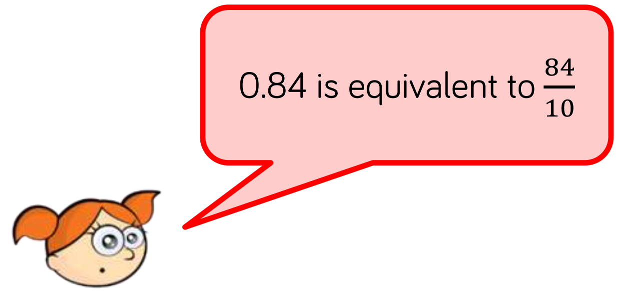
Which is the odd one out and why?

Possible response:

D is the odd one out because it shows 0.3

Explore how the rest represent 0.6

Alex says,



Do you agree?
Explain why.

Possible response:

Alex is wrong because 0.84 is 8 tenths and 4 hundredths and $\frac{84}{10}$ is 84 tenths.

Fractions to Decimals (1)

Notes and Guidance

At this point children should know common fractions, such as thirds, quarters, fifths and eighths, as decimals.

Children explore how finding an equivalent fraction where the denominator is 10, 100 or 1,000 makes it easier to convert from a fraction to a decimal.

They investigate efficient methods to convert fractions to decimals.

Mathematical Talk

How many hundredths are equivalent to one tenth?

How could you convert a fraction to a decimal?

Which is the most efficient method? Why?

Which equivalent fraction would be useful?

Varied Fluency

- Match the fractions to the equivalent decimals.

$\frac{2}{5}$	0.04
$\frac{1}{25}$	0.4
$\frac{1}{4}$	0.25

- Use your knowledge of known fractions to convert the fractions to decimals. Show your method for each one.

$\frac{7}{20}$ $\frac{3}{4}$ $\frac{2}{5}$ $\frac{6}{200}$

- Mo says that $\frac{63}{100}$ is less than 0.65

  Do you agree with Mo?
  Explain your answer.

Fractions to Decimals (1)

Reasoning and Problem Solving

Amir says,

The decimal 0.42 can be read as 'four tenths and two hundredths'.

Teddy says,

The decimal 0.42 can be read as 'forty-two hundredths'.

Who do you agree with? Explain your answer.

Both are correct. Four tenths are equivalent to forty hundredths, plus the two hundredths equals forty-two hundredths.

True or False?

0.3 is bigger than $\frac{1}{4}$

Explain your reasoning.

True because $\frac{1}{4}$ is 25 hundredths and 0.3 is 30 hundredths. Therefore, 0.3 is bigger.

Dora and Whitney are converting $\frac{30}{500}$ into a decimal.

- Dora doubles the numerator and denominator, then divides by 10
- Whitney divides both the numerator and the denominator by 5
- Both get the answer $\frac{6}{100} = 0.06$

Which method would you use to work out each of the following?

$\frac{25}{500}$ $\frac{125}{500}$ $\frac{40}{500}$ $\frac{350}{500}$

Explain why you have used a certain method.

Possible response:

$\frac{25}{500}$ - divide by 5, known division fact.

$\frac{125}{500}$ - double, easier than dividing 125 by 5

$\frac{40}{500}$ - divide by 5, known division fact.

$\frac{350}{500}$ - double, easier than dividing 350 by 5

Fractions to Decimals (2)

Notes and Guidance

It is important that children recognise that $\frac{3}{4}$ is the same as $3 \div 4$. They can use this understanding to find fractions as decimals by then dividing the numerator by the denominator.

In the example provided, we cannot make any equal groups of 5 in the ones column so we have exchanged the 2 ones for 20 tenths. Then we can divide 20 into groups of 5

Mathematical Talk

Do we divide the numerator by the denominator or divide the denominator by the numerator? Explain why.

When do we need to exchange?

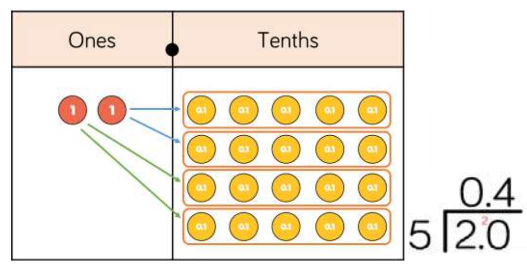
Are we grouping or are we sharing? Explain why.

Why is it useful to write 2 as 2.0 when dividing by 5?

Why is it not useful to write 5 as 5.0 when dividing by 8?

Varied Fluency

- Deena has used place value counters to write $\frac{2}{5}$ as a decimal. She has divided the numerator by the denominator.



Use this method to convert the fractions to decimals. Give your answers to 2 decimal places.

$\frac{1}{2}$ $\frac{3}{4}$

- Use the short division method to convert the fractions to decimals. Write the decimals to three decimal places.

$\frac{5}{8}$ $\frac{4}{5}$ $\frac{8}{5}$

- 8 friends share 7 pizzas.
How much pizza does each person get?
Give your answer as a decimal and as a fraction.

Fractions to Decimals (2)

Reasoning and Problem Solving

Rosie and Tommy have both attempted to convert $\frac{2}{8}$ into a decimal.

I converted $\frac{2}{8}$ into 0.25

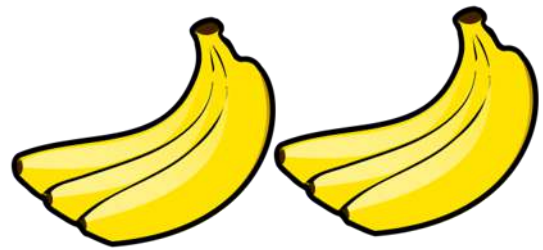
I converted $\frac{2}{8}$ into 4

Who is correct?
Prove it.

Rosie is correct and Tommy is incorrect.

Tommy has divided 8 by 2 rather than 2 divided by 8 to find the answer.

Mo shares 6 bananas between some friends.



Each friend gets 0.75 of a banana.

How many friends does he share the bananas with?
Show your method.

Mo shares his 6 bananas between 8 friends because 6 divided by 8 equals 0.75

Children may show different methods:

Method 1: Children add 0.75 until they reach 6. This may involve spotting that 4 lots of 0.75 equals 3 and then they double this to find 8 lots of 0.75 equals 6

Method 2: Children use their knowledge that 0.75 is equivalent to $\frac{3}{4}$ to find the equivalent fraction of $\frac{6}{8}$

Spring - Block 2

Percentages

Overview

Small Steps

- Understand percentages (R)
- Fractions to percentages
- Equivalent FDP
- Order FDP
- Percentage of an amount (1)
- Percentage of an amount (2)
- Percentages – missing values

Notes for 2020/21

Children should have been introduced to percentages briefly in Y5 but this work may have been missed. Time spent exploring 100 as a denominator, making the link to decimals and hundredths is important.
Bar models and hundred squares should be used to support understanding.

Understand Percentages

Notes and Guidance

Children are introduced to 'per cent' for the first time and will understand that 'per cent' relates to 'number of parts per hundred'.

They will explore this through different representations which show different parts of a hundred. Children will use 'number of parts per hundred' alongside the % symbol.

Mathematical Talk

How many parts is the square split in to?

How many parts per hundred are shaded/not shaded?

Can we represent this percentage differently?

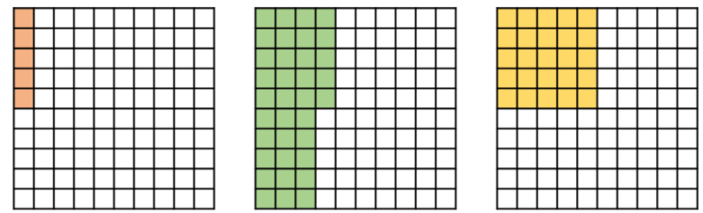
Look at the bar model, how many parts is it split into?

If the bar is worth 100%, what is each part worth?

Varied Fluency

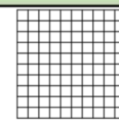
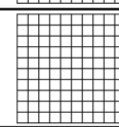
R

Complete the sentence stem for each diagram.

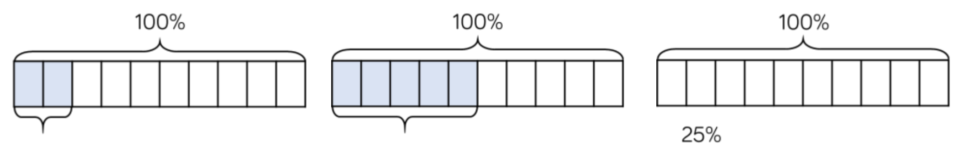


There are ____ parts per hundred shaded. This is ____%

Complete the table.

Pictorial	Parts per hundred	Percentage
	There are 51 parts per hundred.	
		75%

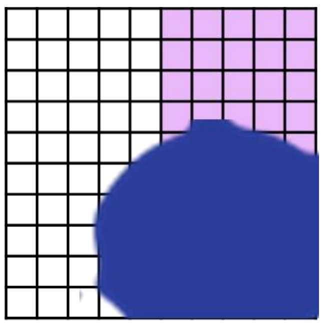
Complete the bar models.



Understand Percentages

Reasoning and Problem Solving

Oh no! Dexter has spilt ink on his hundred square.



Complete the sentence stems to describe what percentage is shaded.

It could be…

It must be…

It can't be…

Some possible answers:

It could be 25%

It must be less than 70%

It can't be 100%

Mo, Annie and Tommy all did a test with 100 questions. Tommy got 6 fewer questions correct than Mo.

Name	Score	Percentage
Mo	56 out of 100	
Annie		65%
Tommy		

Complete the table.
How many more marks did each child need to score 100%?

56%
65 out of 100
50 out of 100
50%

Mo needs 44
Annie needs 35
Tommy needs 50

Dora and Amir each have 100 sweets. Dora eats 65% of hers. Amir has 35 sweets left.
Who has more sweets left?

Neither. They both have an equal number of sweets remaining.

Fractions to Percentages

Notes and Guidance

It is important that children understand that 'percent' means 'out of 100'.
Children will be familiar with converting some common fractions from their work in Year 5
They learn to convert fractions to equivalent fractions where the denominator is 100 in order to find the percentage equivalent.

Mathematical Talk

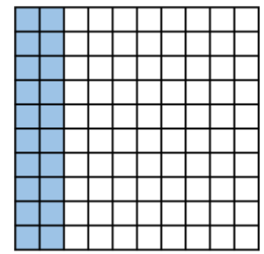
What does the word 'percent' mean?

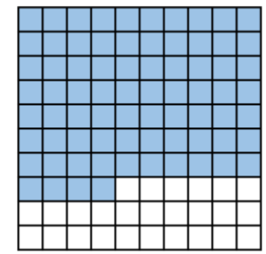
How can you convert tenths to hundredths?

Why is it easy to convert fiftieths to hundredths?
What other fractions are easy to convert to percentages?

Varied Fluency

- What fraction of each hundred square is shaded? Write the fractions as percentages.





- Complete the table.

Fraction	Percentage
$\frac{1}{2}$	
$\frac{1}{4}$	
$\frac{1}{10}$	
$\frac{1}{5}$	

- Fill in the missing numbers.

$\frac{12}{100} = \square\,\%$ $\qquad \frac{\square}{100} = 35\%$

$\frac{12}{50} = \frac{\square}{100} = \square\,\%$ $\qquad \frac{44}{\square} = \frac{22}{100} = 22\%$

Fractions to Percentages

Reasoning and Problem Solving

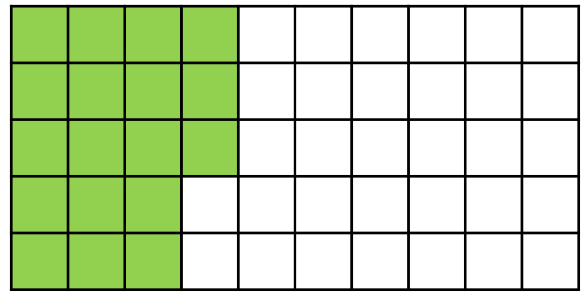
In a Maths test, Tommy answered 62% of the questions correctly.

Rosie answered $\frac{3}{5}$ of the questions correctly.

Who answered more questions correctly?

Explain your answer.

Tommy answered more questions correctly because $\frac{3}{5}$ as a percentage is 60% and this is less than 62%



Amir thinks that 18% of the grid has been shaded.

Dora thinks that 36% of the grid has been shaded.

Who do you agree with?

Explain your reasoning.

Dora is correct because $\frac{18}{50} = \frac{36}{100}$

Equivalent FDP

Notes and Guidance

Children use their knowledge of common equivalent fractions and decimals to find the equivalent percentage.

A common misconception is that 0.1 is equivalent to 1%. Diagrams may be useful to support understanding the difference between tenths and hundredths and their equivalent percentages.

Mathematical Talk

How does converting a decimal to a fraction help us to convert it to a percentage?

How do you convert a percentage to a decimal?

Can you use a hundred square to represent your conversions?

Varied Fluency

Complete the table.

Decimal	Fraction	Percentage
0.35	$\frac{35}{100}$	35%
0.27		
0.6		
0.06		

Use <, > or = to complete the statements.

0.36 ◯ 40%　　　$\frac{7}{10}$ ◯ 0.07

0.4 ◯ 25%　　　0.4 ◯ $\frac{1}{4}$

Which of these are equivalent to 60%?

$\frac{60}{100}$　$\frac{6}{100}$　0.06　$\frac{3}{5}$　$\frac{3}{50}$　0.6

Equivalent FDP

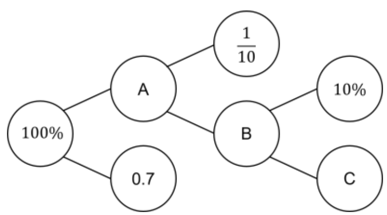
Reasoning and Problem Solving

Amir says 0.3 is less than 12% because 3 is less than 12

Explain why Amir is wrong.

Amir is wrong because 0.3 is equivalent to 30%

Complete the part-whole model.
How many different ways can you complete it?



Can you create your own version with different values?

A = 0.3, 30% or $\frac{3}{10}$

B = 0.2, 20%, $\frac{2}{10}$ or $\frac{1}{5}$

C = 0.1, 10% or $\frac{1}{10}$

How many different fractions can you make using the digit cards?

1 2 3 4 5 6

How many of the fractions can you convert into decimals and percentages?

Possible answers:

Children make a range of fractions.

They should be able to convert $\frac{1}{2}, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}$ and $\frac{4}{5}$ into decimals and percentages.

Order FDP

Notes and Guidance

Children convert between fractions, decimals and percentages to enable them to order and compare them.

Encourage them to convert each number to the same form so that they can be more easily ordered and compared. Once the children have compared the numbers, they will need to put them back into the original form to answer the question.

Mathematical Talk

What do you notice about the fractions, decimals or percentages? Can you compare any straight away?

What is the most efficient way to order them?

Do you prefer to convert your numbers to decimals, fractions or percentages? Why?

If you put them in ascending order, what will it look like?
If you put them in descending order, what will it look like?

Varied Fluency

- Use <, > or = to complete the statements:

$60\% \bigcirc 0.6 \bigcirc \frac{3}{5}$

$0.23 \bigcirc 24\% \bigcirc \frac{1}{4}$

$37.6\% \bigcirc \frac{3}{8} \bigcirc 0.27$

- Order from smallest to largest:

50%	$\frac{2}{5}$	0.45	$\frac{3}{10}$	54%	0.05

- Four friends share a pizza. Whitney eats 35% of the pizza, Teddy eats 0.4 of the pizza, Dora eats 12.5% of the pizza and Alex eats 0.125 of the pizza.

  Write the amount each child eats as a fraction.
  Who eats the most? Who eats the least? Is there any left?

Order FDP

Reasoning and Problem Solving

In his first Geography test, Mo scored 38%

In the next test he scored $\frac{16}{40}$

Did Mo improve his score?

Explain your answer.

Mo improved his score.
$\frac{16}{40}$ is equivalent to 40% which is greater than his previous score of 38%

Which month did Eva save the most money?

Estimate your answer using your knowledge of fractions, decimals and percentages.
Explain why you have chosen that month.

In January, Eva saves $\frac{3}{5}$ of her £20 pocket money.

In February, she saves 0.4 of her £10 pocket money.

In March, she saves 45% of her £40 pocket money.

She saved the most money in March.
Estimates:
Over £10 in January because $\frac{3}{5}$ is more than half.
Under £10 in February because she only had £10 to start with and 0.4 is less than half.
Nearly £20 in March because 45% is close to a half.

Percentage of an Amount (1)

Notes and Guidance

Children use known fractional equivalences to find percentages of amounts.
Bar models and other visual representations may be useful in supporting this e.g. 25% $= \frac{1}{4}$ so we divide into 4 equal parts.
In this step, we focus on 50%, 25%, 10% and 1% only.

Mathematical Talk

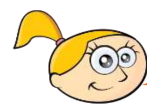
Why do we divide a quantity by 2 in order to find 50%?

How do you calculate 10% of a number mentally?

What's the same and what's different about 10% of 300 and 10% of 30?

Varied Fluency

- Eva says,



50% is equivalent to $\frac{1}{2}$
To find 50% of an amount, I can divide by 2

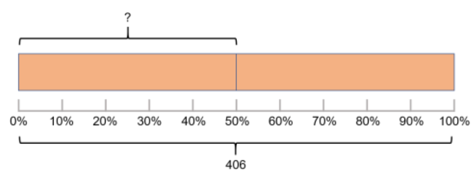
Complete the sentences.

25% is equivalent to $\frac{1}{\square}$ To find 25% of an amount, divide by ___

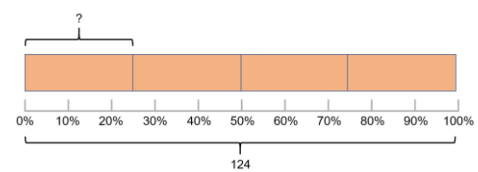
10% is equivalent to $\frac{1}{\square}$ To find 10% of an amount, divide by ___

1% is equivalent to $\frac{1}{\square}$ To find 1% of an amount, divide by ___

- Use the bar models to help you complete the calculations.



50% of 406 =



25% of 124 =

- Find:

50% of 300	25% of 300	10% of 300	1% of 300
50% of 30	25% of 30	10% of 30	1% of 30
50% of 60	25% of 60	10% of 60	1% of 60

Percentage of an Amount (1)

Reasoning and Problem Solving

Mo says, "To find 10% you divide by 10, so to find 50% you divide by 50"  Do you agree? Explain why.	Possible answer: Mo is wrong because 50% is equivalent to a half so to find 50% you divide by 2
Eva says to find 1% of a number, you divide by 100 Whitney says to find 1% of a number, you divide by 10 and then by 10 again. Who do you agree with? Explain your answer.	They are both correct. Whitney has divided by 100 in two smaller steps.

Complete the missing numbers. 50% of 40 = ____% of 80 ___% of 40 = 1% of 400 10% of 500 = ____% of 100	 25 10 50

White Rose Maths

Percentage of an Amount (2)

Notes and Guidance

Children build on the last step by finding multiples of 10% and other known percentages.
They explore different methods of finding certain percentages e.g. Finding 20% by dividing by 10 and multiplying by 2 or by dividing by 5. They also explore finding 5% by finding half of 10%. Using these methods, children build up to find percentages such as 35%.

Mathematical Talk

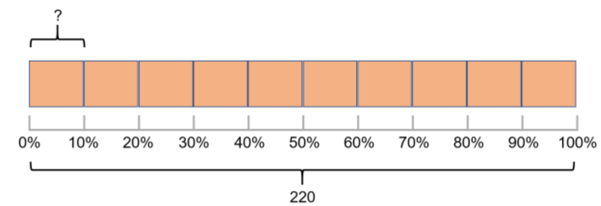
Is dividing by 10 and multiplying by 5 the most efficient way to find 50%? Explain why.

Is dividing by 10 and multiplying by 9 the most efficient way to find 90%? Explain why.

How many ways can you think of to calculate 60% of a number?

Varied Fluency

- Mo uses a bar model to find 30% of 220



10% of 220 = 22 , so 30% of 220 = 3 × 22 = 66

Use Mo's method to calculate:

40% of 220   20% of 110   30% of 440   90% of 460

- To find 5% of a number, divide by 10 and then divide by 2
Use this method to work out:
(a) 5% of 140   (b) 5% of 260   (c) 5% of 1 m 80 cm

How else could we work out 5%?

- Calculate:

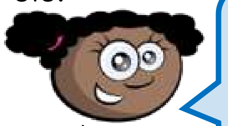
15% of 60 m   35% of 300 g   65% of £20

White Rose Maths

Percentage of an Amount (2)

Reasoning and Problem Solving

Four children in a class were asked to find 20% of an amount, this is what they did:


Whitney

I divided by 5 because 20% is the same as one fifth

I found one percent by dividing by 100, then I multiplied my answer by 20


Amir


Alex

I did 10% add 10%

I found ten percent by dividing by 10, then I multiplied my answer by 2


Jack

Who do you think has the most efficient method? Explain why.
Who do you think will end up getting the answer incorrect?

All methods are acceptable ways of finding 20%. Children may have different answers because they may find different methods easier. Discussion could be had around whether or not their preferred method is always the most efficient.

How many ways can you find 45% of 60?

Use similar strategies to find 60% of 45

What do you notice?

Does this always happen?
Can you find more examples?

Possible methods include:

10% × 4 + 5%

25% + 20%

25% + 10%+ 10%

50% – 5%

To find 60% of 45

10% × 6

50% + 10%

10% × 3

Children will notice that 45% of 60 = 60% of 45

This always happens.

White Rose Maths

Percentages – Missing Values

Notes and Guidance

Children use their understanding of percentages to find the missing whole or a missing percentage when the other values are given. They may find it useful to draw a bar model to help them see the relationship between the given percentage or amount and the whole.

It is important that children see that there may be more than one way to solve a problem and that some methods are more efficient than others.

Mathematical Talk

If we know a percentage, can we work out the whole?

If we know the whole and the amount, can we find what percentage has been calculated?

What diagrams could help you visualise this problem?
Is there more than one way to solve the problem?

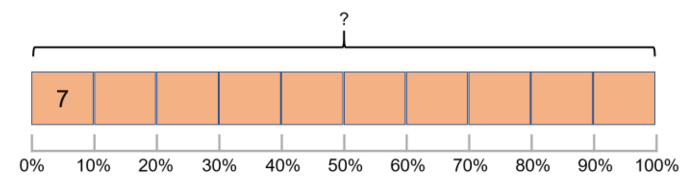
What is the most efficient way to find a missing value?

Varied Fluency

- 350,000 people visited the Natural History Museum last week.
  15% of the people visited on Monday.
  40% of the people visited on Saturday.
  How many people visited the Natural History Museum during the rest of the week?

- If 7 is 10% of a number, what is the number?

  Use the bar model to help you.



- Complete:

  10% of 150 = ☐ 30% of ☐ = 45

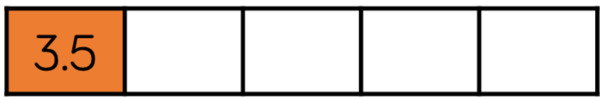
  30% of 300 = ☐ 30% of ☐ = 900

  Can you see a link between the questions?

Percentages – Missing Values

Reasoning and Problem Solving

What percentage questions can you ask about this bar model?



Possible answer:
If 20% of a number is 3.5, what is the whole?
What is 60%?
What is 10%?

Fill in the missing values to make this statement correct.
Can you find more than one way?

25% of ☐ = ☐ % of 60

Possible answers:

25% of 60 = 25% of 60

25% of 120 = 50% of 60

25% of 24 = 10% of 60

25% of 2.4 = 1% of 60

25% of 180 = 75% of 60

A golf club has 200 members.

58% of the members are male.
50% of the female members are children.

(a) How many male members are in the golf club?
(b) How many female children are in the golf club?

.

116 male members

42 female children

Spring - Block 3

Algebra

Overview

Small Steps

- Find a rule – one step
- Find a rule – two step
- Forming expressions
- Substitution
- Formulae
- Forming equations
- Solve simple one-step equations
- Solve two-step equations
- Find pairs of values
- Enumerate possibilities

Notes for 2020/21

All of this block is new learning for Year 6 so there are no recap steps.

Children first look at forming expressions before moving on to solving more complex equations.

This should be introduced using concrete and pictorial methods alongside the abstract notation.

White Rose Maths

Find a Rule – One Step

Notes and Guidance

Children explore simple one-step function machines. Explain that a one-step function is where they perform just one operation on the input.

Children understand that for each number they put into a function machine, there is an output. They should also be taught to "work backwards" to find the input given the output.

Given a set of inputs and outputs, they should be able to work out the function.

Mathematical Talk

What do you think "one-step function" means?

What examples of functions do you know?

Do some functions have more than one name?

What do you think input and output mean?

What is the output if ….?

What is the input if ….?

How many sets of inputs and outputs do you need to be able to work out the function? Explain how you know.

Varied Fluency

Here is a function machine.

Input → $\times 4$ → Output

- What is the output if the input is 2?
- What is the output if the input is 7.2?
- What is the input if the output was 20?
- What is the input if the output was 22?

Complete the table for the function machine.

Input → $+ 5$ → Output

Input	5	5.8	10	– 3	– 8			
Output						9	169	0

Find the missing function.

10 → [ ] → 5

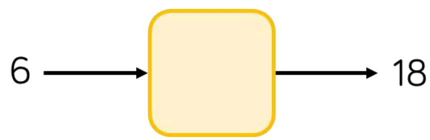
24 → [ ] → 12

7 → [ ] → 3.5

Find a Rule – One Step

Reasoning and Problem Solving

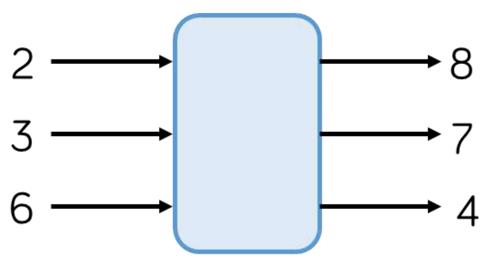
Eva has a one-step function machine. She puts in the number 6 and the number 18 comes out.



What could the function be?
How many different answers can you find?

The function could be $+ 12$, $\times 3$

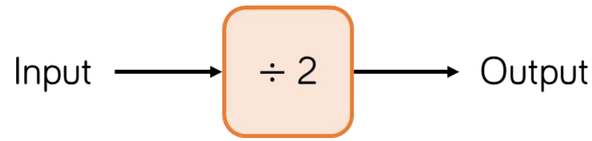
Amir puts some numbers into a function machine.



What is the output from the function when the input is 16?

The function is subtract from 10 so the output is -6

Dora puts a number into the function machine.



Dora's number is:
- A factor of 32
- A multiple of 8
- A square number

What is Dora's input?
What is her output?

Can you create your own clues for the numbers you put into a function machine for a partner to solve?

Dora's input is 16

Her output is 8

White Rose Maths

Find a Rule – Two Step

Notes and Guidance

Children build on their knowledge of one-step functions to look at two-step function machines. Discuss with children whether a function such as + 5 and + 6 is a two-step function machine or whether it can be written as a one-step function.
Children look at strategies to find the functions. They can use trial and improvement or consider the pattern of differences.
Children record their input and output values in the form of a table.

Mathematical Talk

How can you write + 5 followed by – 2 as a one-step function?
If I change the order of the functions, is the output the same?
What is the output if ….?
What is the input if ….?
If you add 3 to a number and then add 5 to the result, how much have you added on altogether?

Varied Fluency

Here is a function machine.

Input → × 2 → + 5 → Output

- What is the output if the input is 5?
- What is the input if the output is 19?
- What is the output if the input is 3.5?

Complete the table for the given function machine.

Input → × 3 → – 4 → Output

Input	1	2	3	4	5
Output					

- What patterns do you notice in the outputs?
- What is the input if 20 is the output? How did you work it out?

How can you write this two-step machine as a one-step machine?

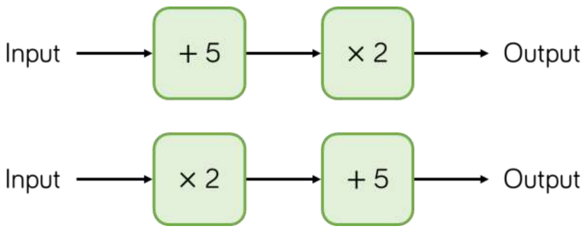
Input → + 3 → + 5 → Output

Check your answer by inputting values.

Find a Rule – Two Step

Reasoning and Problem Solving

Teddy has two function machines.



He says,

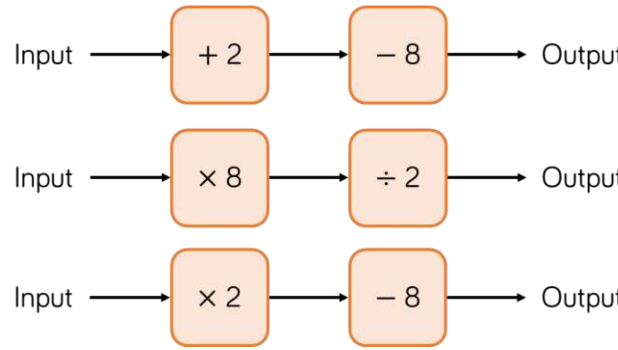
The function machines will give the same answer.

Is Teddy correct?

Is there an input that will give the same output for both machines?

No they do not give the same answer.
Encourage children to refer to the order of operations to help them understand why the outputs are different.

Mo has the following function machines.



Explain which of these can be written as single function machines.

The first one can be written as – 6

The second can be written as × 4

The third cannot be written as a single machine.

Forming Expressions

Notes and Guidance

Children have now met one-step and two-step function machines with numerical inputs.
In this step, children use simple algebraic inputs e.g. y.
Using these inputs in a function machine leads them to forming expressions e.g. $y + 4$. The use of cubes to represent a variable can aid understanding.
Children are introduced to conventions that we use when writing algebraic expressions. e.g. $y \times 4$ as $4y$.

Mathematical Talk

What expressions can be formed from this function machine?

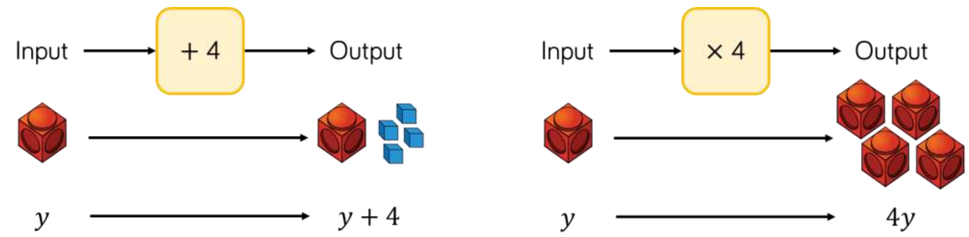
What would the function machine look like for this rule/expression?

How can you write $x \times 3 + 6$ differently?

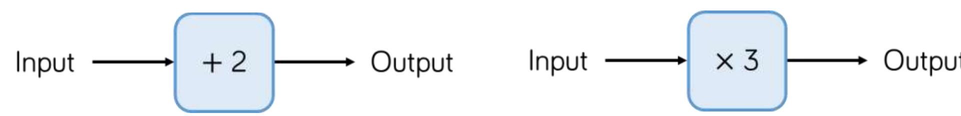
Are $2a + 6$ and $6 + 2a$ the same? Explain your answer

Varied Fluency

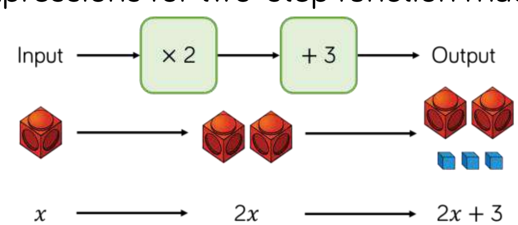
Mo uses cubes to write expressions for function machines.



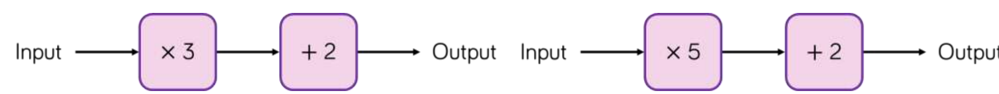
Use Mo's method to represent the function machines.
What is the output for each machine when the input is a?



Eva is writing expressions for two-step function machines.



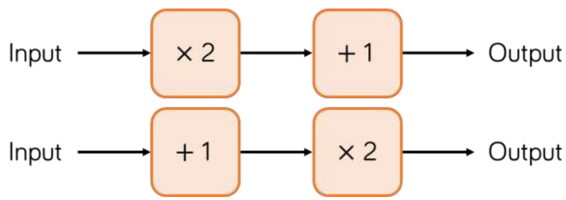
Use Eva's method to write expressions for the function machines.



Forming Expressions

Reasoning and Problem Solving

Amir inputs m into these function machines.



He says the outputs of the machines will be the same.

Do you agree?

Explain your answer.

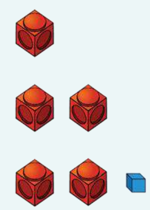
No, because $2m + 1$ isn't the same as $2m + 2$

<u>$2m + 1$</u>

Input

× 2

+ 1

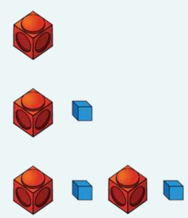


<u>$2m + 2$</u>

Input

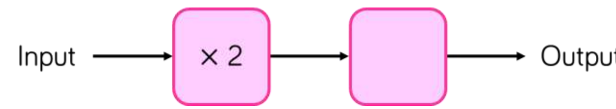
+ 1

× 2



Children may use examples with numbers to show this.

This function machine gives the same output for every input.
For example if the input is 5 then the output is 5 and so on.



What is the missing part of the function?

What other pairs of functions can you think that will do the same?

÷ 2

Other pairs of functions that will do the same are functions that are the inverse of each other e.g. $+ 3, - 3$

Substitution

Notes and Guidance

Children substitute into simple expressions to find a particular value.

They have already experienced inputting into a function machine, and teachers can make the links between these two concepts.

Children will need to understand that the same expression can have different values depending on what has been substituted.

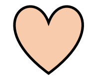
Mathematical Talk

Which letter represents the star?
Which letter represents the heart?
Would it still be correct if it was written as $a + b + c$?

What does it mean when a number is next to a letter?

Is $a + b + b$ the same as $a + 2b$?

Varied Fluency

- If  = 7 and  = 5, what is the value of:

If $a = 7$ and $b = 5$ what is the value of:

$$a + b + b$$

What is the same and what is different about this question?

- Substitute the following to work out the values of the expressions.

$w = 3 \quad x = 5 \quad y = 2.5$

  - $w + 10$
  - $w + x$
  - $y - w$

- Substitute the following to work out the values of the expressions.

$w = 10 \quad x = \frac{1}{4} \quad y = 2.5$

  - $3y$
  - wx
  - $12 + 8.8w$
  - $wy + 4x$

Substitution

Reasoning and Problem Solving

Here are two formulae.

$$p = 2a + 5$$

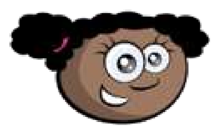
$$c = 10 - p$$

Find the value of c when $a = 10$

$c = -15$

$$x = 2c + 6$$

Whitney says,



$x = 12$ because c must be equal to 3 because it's the 3rd letter in the alphabet

Is Whitney correct?

No Whitney is incorrect. c could have any value.

Amir says,

When $c = 5$, $x = 31$

Amir is wrong.
Explain why.
What would the correct value of x be?

Amir has put the 2 next to the 5 to make 25 instead of multiplying 2 by 5

The correct value of x would be 16

Formulae

Notes and Guidance

Children substitute into familiar formulae such as those for area and volume.

They also use simple formulae to work out values of everyday activities such as the cost of a taxi or the amount of medicine to take given a person's age.

Mathematical Talk

What tells you something is a formula?

Which of the rectangles is the odd one out? Why?

Could you write the formula for a rectangle in a different way?

What other formulae do you know?

Varied Fluency

Which of the following is a formula?

$P = 2l + 2w$ | $3d + 5$ | $20 = 3x - 2$

Explain why the other two are not formulae.

Eva uses the formula $P = 2l + 2w$ to find the perimeter of rectangles.
Use this formula to find the perimeter of rectangles with the following lengths and widths.

- $l = 15, w = 4$
- $l = \frac{1}{4}, w = \frac{3}{8}$
- $l = w = 5.1$

This is the formula to work out the cost of a taxi.

$$C = 1.50 + 0.3m$$

$C =$ the cost of the journey in £
$m =$ number of miles travelled.
Work out the cost of a 12-mile taxi journey

Formulae

Reasoning and Problem Solving

Jack and Dora are using the following formula to work out what they should charge for four hours of cleaning.

Cost in pounds = 20 + 10 × number of hours

Jack thinks they should charge £60
Dora thinks they should charge £120

Who do you agree with?
Why?

Jack is correct as multiplication should be performed first following the order of operations.

Dora has not used the order of operations – she has added 20 and 10 and then multiplied 30 by 4

The rule for making scones is use 4 times as much flour (f) as butter (b).

Which is the correct formula to represent this?

A $f = \frac{b}{4}$

B $f = 4b$

C $f = b + 4$

D $4f = b$

Explain why the others are incorrect.

B is correct.

A shows the amount of flour is a quarter of the amount of butter.

C shows the amount of flour is 4 more than butter.

D shows butter is 4 times the amount of flour.

Forming Equations

Notes and Guidance

Building on the earlier step of forming expressions, children now use algebraic notation to form one-step equations.
They need to know the difference between an expression like $x + 5$, which can take different values depending on the value of x, and an equation like $x + 5 = 11.2$ where x is a specific unknown value. This is best introduced using concrete materials e.g. cubes, can be used to represent the unknown values with counters being used to represent known numbers.

Mathematical Talk

What does the cube represent?
What do the counters represent?

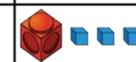
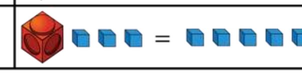
Design your own 'think of a number' problems.

What's the difference between an expression and an equation?

What's the difference between a formula and an equation?

Varied Fluency

Amir represents a word problem using cubes, counters and algebra.

Words	Concrete	Algebra
I think of a number		x
Add 3		$x + 3$
My answer is 5		$x + 3 = 5$

Complete this table using Amir's method.

Words	Concrete	Algebra
I think of a number		
Add 1		
My answer is 8		

A book costs £5 and a magazine costs £n
The total cost of the book and magazine is £8
Write this information as an equation.

Write down algebraic equations for these word problems.
- I think of a number, subtract 17, my answer is 20
- I think of a number, multiply it by 5, my answer is 45

Forming Equations

Reasoning and Problem Solving

Rosie thinks of a number. She adds 7 and divides her answer by 2 Teddy thinks of a number. He multiples by 3 and subtracts 4 Rosie and Teddy think of the same number. Rosie's answer is 9 What is Teddy's answer?	They both think of 11, therefore Teddy's answer is 29
Rosie and Teddy think of the same number again. This time, they both get the same answer. Use trial and improvement to find the number they were thinking of.	They think of 3 and the answer they both get is 5

Eva spends 92p on yo-yos and sweets She buys y yo-yos costing 11p and s sweets costing 4p. Can you write an equation to represent what Eva has bought? How many yo-yos and sweets could Eva have bought?	$92 = 11y + 4s$ She could have bought 1 sweet and 8 yo-yos or 4 yo-yos and 12 sweets.
Can you write a similar word problem to describe this equation? $74 = 15t + 2m$	

One-step Equations

Notes and Guidance

Children solve simple one step equations involving the four operations.

Children should explore this through the use of concrete materials such as cubes, counters and cups.

It is recommended that children learn to solve equations using a balancing method using inverse operations.

Mathematical Talk

Can you make some of your own equations using cups and counters for a friend to solve?

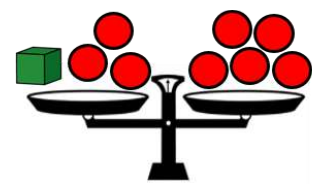
Why do you think the equation is set up on a balance? What does the balance represent? How does this help you solve the equation?

What is the same and what is different about each bar model?

Varied Fluency

- How many counters is each cup worth?
  Write down and solve the equation represented by the diagram.

- Solve the equation represented on the scales.
  Can you draw a diagram to go with the next step?

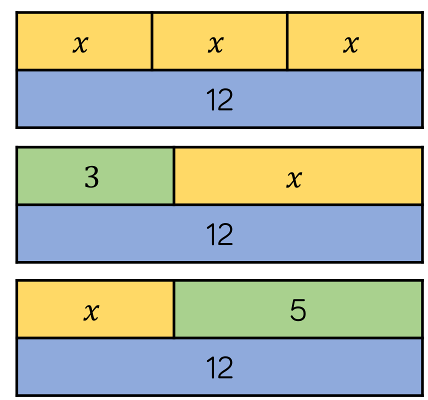


- Match each equation to the correct bar model and then solve to find the value of x.

$x + 5 = 12$

$3x = 12$

$12 = 3 + x$

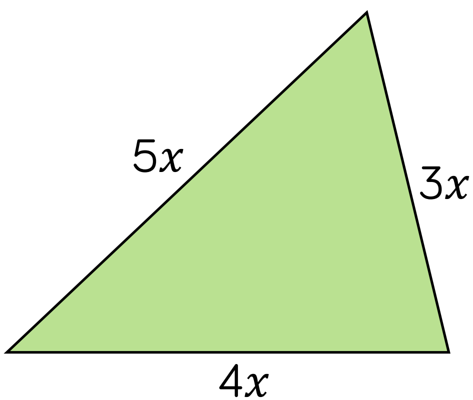


White Rose Maths

One-step Equations

Reasoning and Problem Solving

The perimeter of the triangle is 216 cm.



Form an equation to show this information.

Solve the equation to find the value of x.

Work out the lengths of the sides of the triangle.

$3x + 4x + 5x = 216$
$12x = 216$
$x = 18$

$5 \times 18 = 90$
$3 \times 18 = 54$
$4 \times 18 = 72$

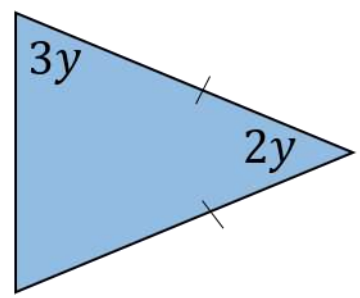
- Hannah is 8 years old
- Jack is 13 years old
- Grandma is $x + 12$ years old.
- The sum of their ages is 100

Form and solve an equation to work out how old Grandma is.

$8 + 13 + x + 12 = 100$

$33 + x = 100$
$x = 77$
Grandma is 77 years old.

What is the size of the smallest angle in this isosceles triangle?



How can you check your answer?

$8y = 180$
$y = 22.5$

Smallest angle = 45°
Check by working them all out and see if they add to 180°

Two-step Equations

Notes and Guidance

Children progress from solving equations that require one-step to equations that require two steps.
Children should think of each equation as a balance and solve it through doing the same thing to each side of the equation.
This should be introduced using concrete and pictorial methods alongside the abstract notation as shown. Only when secure in their understanding should children try this without the support of bar models or similar representations.

Mathematical Talk

Why do you have to do the same to each side of the equation?

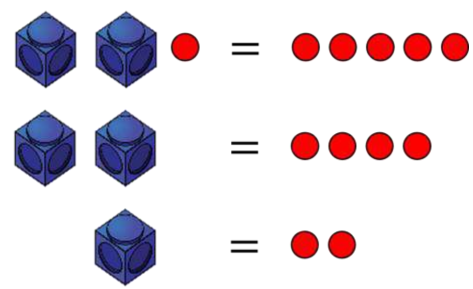
Why subtract 1? What does this do to the left hand side of the equation?

Does the order the equation is written in matter?

What's the same and what's different about solving the equations $2x + 1 = 17$ and $2x - 1 = 17$?

Varied Fluency

Here is each step of an equation represented with concrete resources.



$2x + 1 = 5$

-1 $\quad$ -1

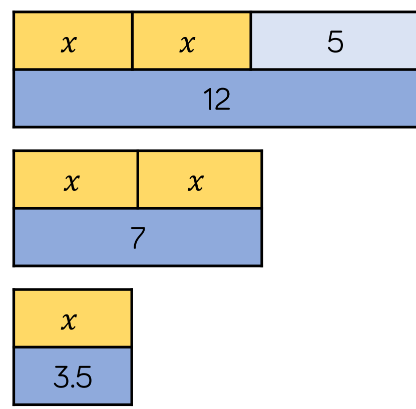
$2x = 4$

$\div 2$ $\quad$ $\div 2$

$x = 2$

Use this method to solve:

$4y + 2 = 6$ $\qquad$ $9 = 2x + 5$ $\qquad$ $1 + 5a = 16$



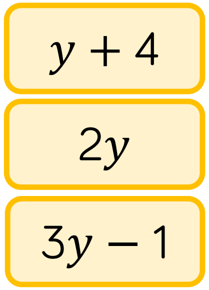
Here is each step of an equation represented by a bar model.
Write the algebraic steps that show the solution of the equation.

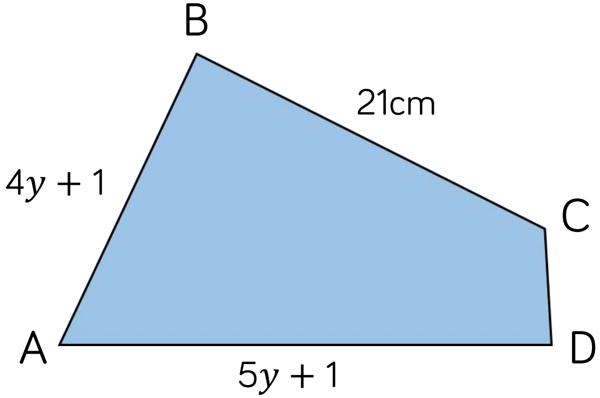
Use bar models to solve these equations.

$3b + 4 = 19$ $\qquad$ $20 = 4b + 2$

Two-step Equations

Reasoning and Problem Solving

The length of a rectangle is $2x + 3$ The width of the same rectangle is $x - 2$ The perimeter is 17 cm. Find the area of the rectangle.	$6x + 2 = 17$ $6x = 15$ $x = 2.5$ Length = 8 cm Width = 0.5 cm Area = 4 cm^2
Alex has some algebra expression cards.   The mean of the cards is 19 Work out the value of each card.	$6y + 3 = 57$ $6y = 54$ $y = 9$ Card values: 13 18 26

Here is the quadrilateral ABCD. The perimeter of the quadrilateral is 80 cm.  AB is the same length as BC. Find the length of CD.	$4y + 1 = 21$ $4y = 20$ $y = 5$ AB = 21 cm BC = 21 cm AD = 26 cm CD = 80 – (21 + 21 + 26) = 12 cm

Find Pairs of Values (1)

Notes and Guidance

Children use their understanding of substitution to consider what possible values a pair of variables can take.

At this stage we should focus on integer values, but other solutions could be a point for discussion.

Children can find values by trial and improvement, but should be encouraged to work systematically.

Mathematical Talk

Can a and b be the same value?

Is it possible for a or b to be zero?

How many possible integer answers are there? Convince me you have them all.

What do you notice about the values of c and d?

Varied Fluency

a and b are variables:

$$a + b = 6$$

There are lots of possible solutions to This equation.
Find 5 different possible integer values for a and b.

a	b

X and Y are whole numbers.

- X is a one digit odd number.
- Y is a two digit even number.
- $X + Y = 25$

Find all the possible pairs of numbers that satisfy the equation.

$$c \times d = 48$$

What are the possible integer values of c and d?
How many different pairs of values can you find?

Find Pairs of Values (1)

Reasoning and Problem Solving

a, b and c are integers between 0 and 5

$$a + b = 6$$
$$b + c = 4$$

Find the values of a, b and c

How many different possibilities can you find?

Possible answers:

$a = 4$ $b = 2$ $c = 2$

$a = 3$ $b = 3$ $c = 1$

$a = 2$ $b = 4$ $c = 0$

x and y are both positive whole numbers.

$$\frac{x}{y} = 4$$

Dora says,

x will always be a multiple of 4

Jack says,

y will always be a factor of 4

Only one is correct – who is it? Explain your answer.

Possible answer:

Dora is correct as x will always have to divide into 4 equal parts e.g. $32 \div 8 = 4$, $16 \div 4 = 4$

Jack is incorrect. $40 \div 10 = 4$ and 10 is not a factor of 4

Find Pairs of Values (2)

Notes and Guidance

Building on from the last step, children find possible solutions to equations which involve multiples of one or more unknown.

They should be encouraged to try one number for one of the variables first and then work out the corresponding value of the other variable. Children should then work systematically to test if there are other possible solutions that meet the given conditions.

Mathematical Talk

What does $2a$ mean? (2 multiplied by an unknown number)
What is the greatest/smallest number 'a' can be?

What strategy did you use to find the value of 'b'?

Can you draw a bar model to represent the following equations:
$3f + g = 20$
$7a + 3b = 40$
What could the letters represent?

Varied Fluency

- In this equation, a and b are both whole numbers which are less than 12.

$$2a = b$$

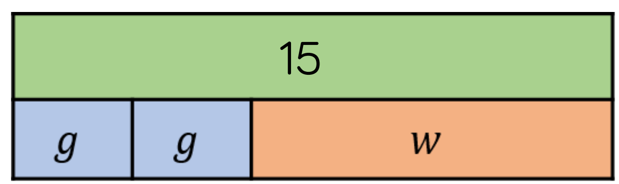
Write the calculations that would show all the possible values for a and b.

- Chose values of x and use the equation to work out the values of y.

$$7x + 4 = y$$

Value of x	Value of y

- $2g + w = 15$
g and w are positive whole numbers.
Write down all the possible values for g and w, show each of them in a bar model.

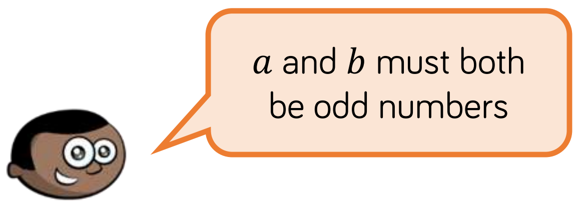


Find Pairs of Values (2)

Reasoning and Problem Solving

$ab + b = 18$

Mo says,



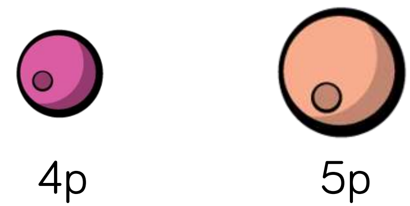
Is Mo correct?
Explain your answer.

Possible answer:

Mo is incorrect. Children may give examples to prove Mo is correct e.g. if $a = 5$ and $b = 3$, but there are also examples to show he is incorrect e.g. $a = 2$ and $b = 6$ where a and b are both even.

Large beads cost 5p and small beads cost 4p

Rosie has 79p to spend on beads.



How many different combinations of small and large beads can Rosie buy?

Can you write expressions that show all the solutions?

Possible answers:

$3l + 16s$
$7l + 11s$
$11l + 6s$
$15l + s$

Spring - Block 4

Converting Units

Overview

Small Steps

Notes for 2020/21

- Metric measures
- Convert metric measures
- Calculate with metric measures
- Miles and kilometres
- Imperial measures

All of this block is new learning for Year 6 so there are no recap steps.

Children explore measures in context and build on previous learning about place value.

Metric Measures

Notes and Guidance

Children read, write and recognise all metric measures for length, mass and capacity. They may need to be reminded the difference between capacity (the amount an object can contain) and volume (the amount actually in an object).

They develop their estimation skills in context and decide when it is appropriate to use different metric units of measure.

Mathematical Talk

Which units measure length? Mass? Capacity?

When would you use km instead of m? When would you use mm instead of cm?

Which is the most appropriate unit to use to measure the object? Explain your answer.

Why do you think _____ is not an appropriate estimate?

Varied Fluency

Choose the unit of measure that would be the most appropriate to measure the items.

cm kg km g tonnes ml mm litres

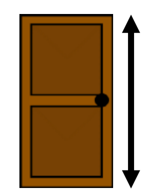
- The weight of an elephant
- The volume of water in a bath
- The length of an ant
- The length of a football pitch
- The weight of an apple

Estimate how much juice the glass holds:

250 ml 2 litres 0.5 litres $\frac{1}{2}$ kg

Estimate the height of the door frame:

20 mm 20 cm 20 m 2 km 2 m 0.2 km

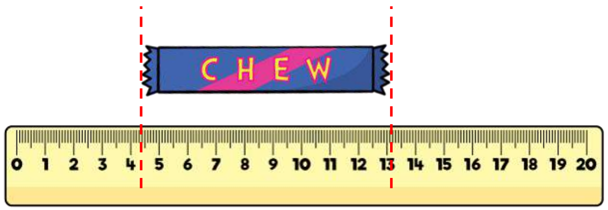


Metric Measures

Reasoning and Problem Solving

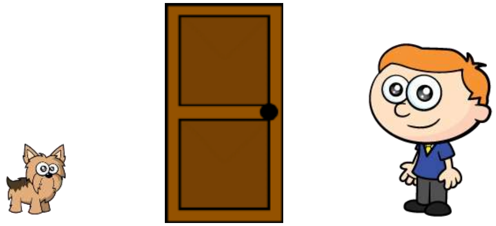
Teddy thinks his chew bar is 13.2 cm long.

Do you agree? Explain why.



Teddy is wrong because he has not lined up the end of his chew bar with zero.
It is actually 8.8 cm long.

Ron's dog is about $\frac{1}{4}$ of the height of the door.
Ron is three times the height of his dog.
Estimate the height of Ron and his dog.



Door = 2 m (200 cm)
Dog = 50 cm
Ron = 150 cm

Here is a train timetable showing the times of trains travelling from Halifax to Leeds.

Halifax	Leeds
07:33	08:09
07:49	08:37
07:52	08:51

An announcement states all trains will arrive $\frac{3}{4}$ of an hour late.
Which train will arrive in Leeds closest to 09:07?

The first train from Halifax, which will now arrive in Leeds at 08:54.

Convert Metric Measures

Notes and Guidance

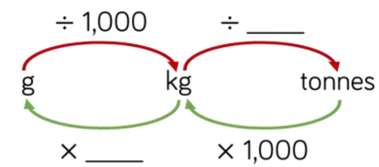
Children will use their skills of multiplying and dividing by 10, 100 and 1,000 when converting between units of length, mass and capacity.
Children will convert in both directions e.g. m to cm and cm to m. Using metre sticks and other scales will support this step.
They will need to understand the role of zero as a place holder when performing some calculations, as questions will involve varied numbers of decimal places.

Mathematical Talk

How could you work out what each mark is worth on the scales?
What do you think would be the most efficient method for converting the units of time?
What's the same and what's different between 1.5 km and 1.500 km? Are the zeroes needed? Why or why not?
What do you notice about the amounts in the table? Can you spot a pattern?
What's the same and what's different about km and kg?

Varied Fluency

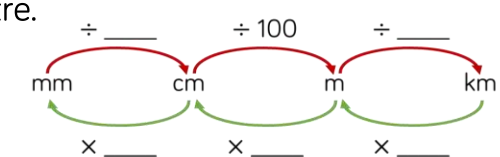
There are ___ grams in one kilogram.
There are ___ kilograms in one tonne.
Use these facts to complete the tables.



g	kg
1,500	
	2.05
1,005	

kg	tonnes
1,202	
	4.004
125	

There are ___ mm in one centimetre.
There are ___ cm in one metre.
There are ___ m in one kilometre.
Use these facts to complete the table.



mm	cm	m	km
44,000			
	2,780		
		15.5	
			1.75

Convert Metric Measures

Reasoning and Problem Solving

Mo thinks that 12,000 g is greater than 20 kg because 12,000 > 20 Explain why Mo is wrong.	12,000 g = 12 kg, which is less than 20 kg.
Put these capacities in order, starting with the smallest. 3 litres 3,500 ml 0.4 litres 0.035 litres 450 ml 330 ml	0.035 litres 330 ml 0.4 litres 450 ml 3 litres 3,500 ml
A shop sells one-litre bottles of water for 99p each. 300 ml bottles of water are on offer at 8 bottles for £2 Whitney wants to buy 12 litres of water. Find the cheapest way she can do this.	£11.88 to buy 12 one-litre bottles. 12 litres = 40 bottles of size 300 ml. $40 \div 8 = 5$ so this will cost $5 \times 2 = £10$ Whitney should buy 40 bottles of 300 ml.

Calculate with Metric Measures

Notes and Guidance

Children use and apply their conversion skills to solve measurement problems in context.

Teachers should model the use of pictorial representations, such as bar models, to represent the problem and help them decide which operation to use.

Mathematical Talk

What operation are you going to use and why?

How could you use a bar model to help you understand the question?

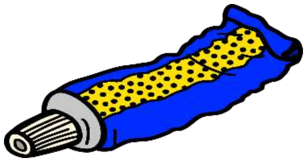
How many ___ are there in a ___?

How can we convert between ___ and ___?

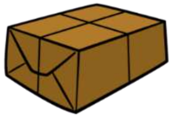
Varied Fluency

- A tube of toothpaste holds 75 ml.

  How many tubes can be filled using 3 litres of toothpaste?



- A parcel weighs 439 grams. How much would 27 parcels weigh? Give your answer in kilograms.



- To bake buns for a party, Ron used these ingredients:

  600 g caster sugar
  0.6 kg butter
  18 eggs (792 g)
  $\frac{3}{4}$ kg self-raising flour
  10 g baking powder

  What is the total mass of the ingredients? Give your answer in kilograms.

Calculate with Metric Measures

Reasoning and Problem Solving

Jack, Alex and Amir jumped a total of 12.69 m in a long jump competition. Alex jumped exactly 200 cm further than Jack. Amir jumped exactly 2,000 mm further than Alex. What distance did they all jump? Give your answers in metres.	Jack jumped 2.23 m. Alex jumped 4.23 m. Amir jumped 6.23 m.
Dora made a stack of her magazines. Each magazine on the pile is 2.5 mm thick. The total height of the stack is 11.5 cm high. How many magazines does she have in her pile?	There are 46 magazines in Dora's pile.

Each nail weighs 3.85 grams.  There are 24 nails in a packet. What would be the total mass of 60 packets of nails? Give your answer in kilograms.	5.544 kg
How many packets would you need if you wanted $\frac{1}{2}$ kg of nails?	6 packets (554.4 g)
How many grams of nails would be left over?	55.4 g left over

White Rose Maths

Miles and Kilometres

Notes and Guidance

Children need to know that 5 miles is approximately equal to 8 km. They should use this fact to find approximate conversions from miles to km and from km to miles.

They should be taught the meaning of the symbol '≈' as "is approximately equal to".

Mathematical Talk

Give an example of a length you would measure in miles or km.

If we know 5 miles ≈ 8 km, how can we work out 15 miles converted to km?

Can you think of a situation where you may need to convert between miles and kilometres?

Varied Fluency

5 miles ≈ 8 kilometres

Use this fact to complete:

- 15 miles ≈ _______ km
- 30 miles ≈ _______ km
- _______ miles ≈ 160 km

If 10 miles is approximately 16 km, 1 mile is approximately how many kilometres?

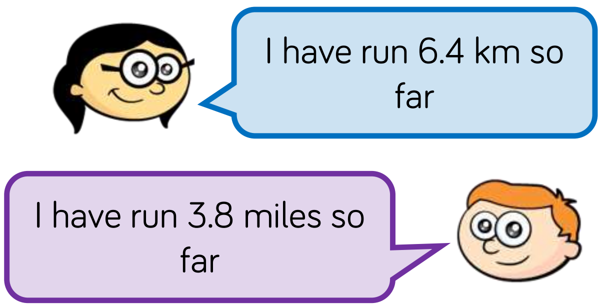
- 2 miles ≈ _______ km
- 4 miles ≈ _______ km
- 0.5 miles ≈ _______ km

In the United Kingdom, the maximum speed on a motorway is 70 miles per hour (mph).
In France, the maximum speed on a motorway is 130 kilometres per hour (km/h).
Which country has the higher speed limit, and by how much? Give your answer in both units.

Miles and Kilometres

Reasoning and Problem Solving

Ron and Annie are running a 5 mile race.



Who has the furthest left to run?

Annie has 1 mile left to run, whereas Ron has 1.2 miles left to run.
Ron has the furthest left to run.

The distance between Cardiff and London is 240 km.

A car is travelling at 60 mph.

How long will it take them to get to London from Cardiff?

240 km ≈ 150 miles
$150 \div 60 = 2\frac{1}{2}$ hours
Or
60 miles ≈ 96 km
$240 \div 96 = 2\frac{1}{2}$ hours

Mo cycles 45 miles over the course of 3 days.

On day 1, he cycles 16 km.

On day 2, he cycles 10 miles further than he did on day 1

How far does he cycle on day 3?

Give your answer in miles and in kilometres.

On day 1 he cycles 16 km / 10 miles.

On day 2 he cycles 32 km / 20 miles.

On day 3 he cycles 24 km / 15 miles.

White Rose Maths

Imperial Measures

Notes and Guidance

Children need to know and use the following facts:

- 1 foot is equal to 12 inches
- 1 pound is equal to 16 ounces
- 1 stone is equal to 14 pounds
- 1 gallon is equal to 8 pints
- 1 inch is approximately 2.5 cm

They should use these to perform related conversions, both within imperial measures and between imperial and metric.

Mathematical Talk

Put these in order of size: 1 cm, 1 mm, 1 inch, 1 foot, 1 metre. How do you know?

When do we use imperial measures instead of metric measures?

Why are metric measures easier to convert than imperial measures?

Varied Fluency

2.5 cm ≈ 1 inch	1 foot = 12 inches

Use these facts to complete:

2 feet = ____ inches   6 inches ≈ ____ cm

____ feet = 36 inches   4 feet ≈ ____ cm

1 pound (lb) = 16 ounces	1 stone = 14 pounds (lbs)

Use this fact to complete:

2 lbs = ____ ounces   5 stone = ____ lbs

____ lbs = 320 ounces   ____ stones = 154 lbs

1 gallon = 8 pints

- How many gallons are equivalent to 64 pints?
- How many pints are equivalent to 15 gallons?
- How many gallons are equivalent to 2 pints?

Imperial Measures

Reasoning and Problem Solving

Jack is 6 foot 2 inches tall. Rosie is 162 cm tall. Who is taller and by how much?	Jack is 185 cm tall, he is 23 cm taller than Rosie.
60 gallons of water are drunk at a sports day. Each child drank 3 pints. How many children were at the sports day?	60 gallons = 480 pints 480 ÷ 3 = 160 children

Eva wants to make a cake. Here are some of the ingredients she needs: • 8 ounces of caster sugar • 6 ounces of self-raising flour • 6 ounces of butter This is what Eva has in her cupboards: • 0.5 lbs of caster sugar • 0.25 lbs of self-raising flour • $\frac{3}{8}$ lbs of butter Does Eva have enough ingredients to bake the cake? If not, how much more does she need to buy? 	Eva has the exact amount of butter and caster sugar, but does not have enough self-raising flour – she needs another 2 ounces.

White Rose Maths

Spring - Block 5

Area, Perimeter & Volume

Overview

Small Steps

- Shapes – same area
- Area and perimeter
- Area of a triangle (1)
- Area of a triangle (2)
- Area of a triangle (3)
- Area of parallelogram
- What is volume? (R)
- Volume – counting cubes
- Volume of a cuboid

Notes for 2020/21

Much of this block is new learning where children build on their knowledge of area and perimeter to explore the area of a triangles and parallelograms.

The recap step on volume covers the difference between volume and capacity and gives time to explore the conservation of volume using centimetre cubes.

Shapes – Same Area

Notes and Guidance

Children will find and draw rectilinear shapes that have the same area.

Children will use their knowledge of factors to draw rectangles with different areas. They will make connections between side lengths and factors.

Mathematical Talk

What do we need to know in order to work out the area of a shape?

Why is it useful to know your times-tables when calculating area?

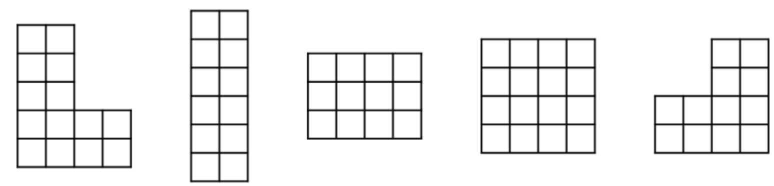
Can you have a square with an area of 48 cm^2? Why?

How can factors help us draw rectangles with a specific area?

Varied Fluency

- Sort the shapes into the Carroll diagram.

	Quadrilateral	Not a quadrilateral
Area of 12 cm^2		
Area of 16 cm^2		



Now draw another shape in each section of the diagram.

- How many rectangles can you draw with an area of 24 cm^2 where the side lengths are integers?

  What do you notice about the side lengths?

  Using integer side lengths, draw as many rectangles as possible that give the following areas:

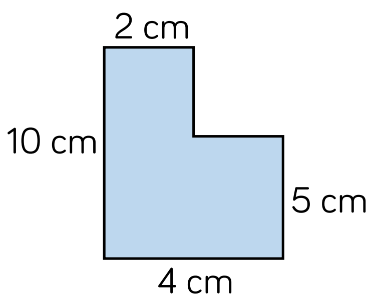
  17 cm^2 25 cm^2 32 cm^2

Shapes – Same Area

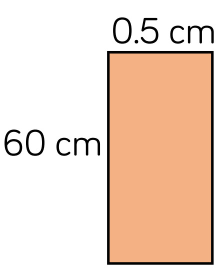
Reasoning and Problem Solving

Rosie and Dexter are drawing shapes with an area of $30cm^2$

Rosie's shape



Dexter's shape



Who is correct?

Explain your reasoning.

Both are correct.

Dexter's shape:
60 cm × 0.5 cm
= 30 cm^2

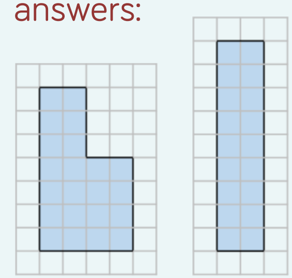
Rosie's shape:
2 cm × 10 cm
= 20 cm^2
5 cm × 2 cm
= 10 cm^2
20 cm^2 + 10 cm^2
= 30 cm^2
Could be split differently.

Three children are given the same rectilinear shape to draw.

Amir says, "The smallest length is 2 cm."
Alex says, "The area is less than 30 cm^2."
Annie says, "The perimeter is 22 cm."

What could the shape be?
How many possibilities can you find?

Children can use squared paper to explore. Possible answers:



Always, Sometimes, Never?

If the area of a rectangle is odd then all of the lengths are odd.

Sometimes – 15 cm^2 could be 5 cm and 3 cm or 60 cm and 0.25 cm

Area and Perimeter

Notes and Guidance

Children should calculate area and perimeter of rectilinear shapes. They must have the conceptual understanding of the formula for area by linking this to counting squares. Writing and using the formulae for area and perimeter is a good opportunity to link back to the algebra block.
Children explore that shapes with the same area can have the same or different perimeters.

Mathematical Talk

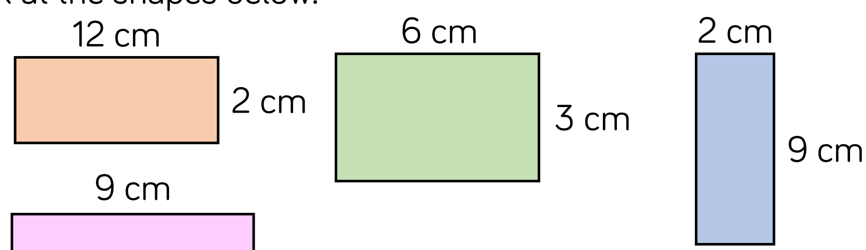
What is the difference between the area and perimeter of a shape?

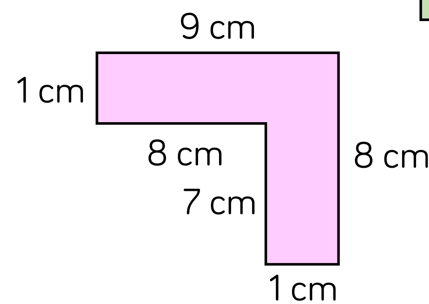
How do we work out the area and perimeter of shapes? Can you show this as a formula?

Can you have 2 rectangles with an area of 24 cm² but different perimeters?

Varied Fluency

- Look at the shapes below.





Do any of the shapes have the same area?

Do any of the shapes have the same perimeter?

- Work out the missing values.

- Draw two rectilinear shapes that have an area of 36 cm² but have different perimeters.

  What is the perimeter of each shape?

Area and Perimeter

Reasoning and Problem Solving

True or false?

Two rectangles with the same perimeter can have different areas.

Explain your answer.

True. Children explore this by drawing rectangles and comparing both area and perimeter.

A farmer has 60 metres of perimeter fencing.

For every 1 m^2 he can keep 1 chicken.

How can he arrange his fence so that the enclosed area gives him the greatest area?

The greatest area is a 15 m × 15 m square, giving 225 m^2

Children may create rectangles by increasing one side by 1 unit and decreasing one side by 1 unit e.g.

16 × 14 = 224 m^2

17 × 13 = 221 m^2

Tommy has a 8 cm × 2 cm rectangle. He increases the length and width by 1 cm.

Length	Width	Area
8	2	
9	3	

He repeats with a 4 cm × 6 cm rectangle.

Length	Width	Area
4	6	

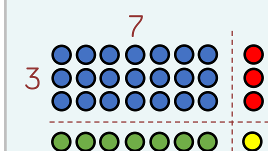
What do you notice happens to the areas?

Can you find any other examples that follow this pattern?

Are there any examples that do not follow the pattern?

If the sum of the length and width is 10, then the area will always increase by 11

Children may use arrays to explore this:



The red and green will always total 10 and the yellow will increase that by 1 to 11

Area of a Triangle (1)

Notes and Guidance

Children will use their previous knowledge of approximating and estimating to work out the area of different triangles by counting.

Children will need to physically annotate to avoid repetition when counting the squares.

Children will begin to see the link between the area of a triangle and the area of a rectangle or square.

Mathematical Talk

How many whole squares can you see?

How many part squares can you see?

What could we do with the parts?

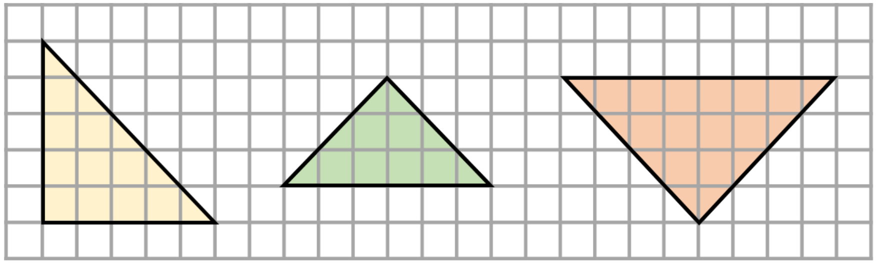
What does estimate mean?

Why is your answer to this question an **estimate** of the area?

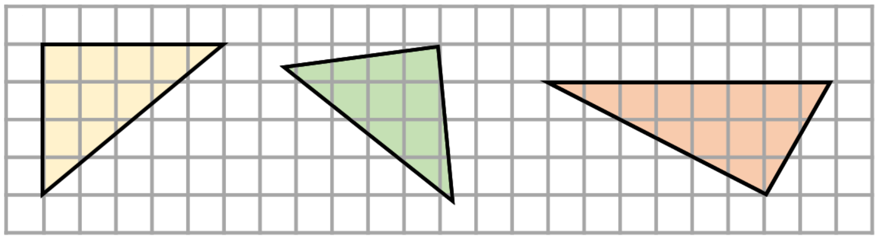
Revisit the idea that a square is a rectangle when generalising how to calculate the area of a triangle.

Varied Fluency

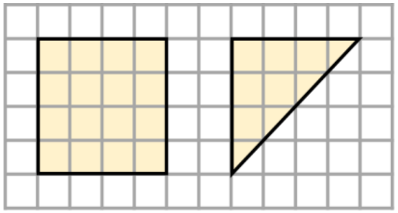
- Count squares to calculate the area of each triangle.



- Estimate the area of each triangle by counting squares.



- Calculate the area of each shape by counting squares.

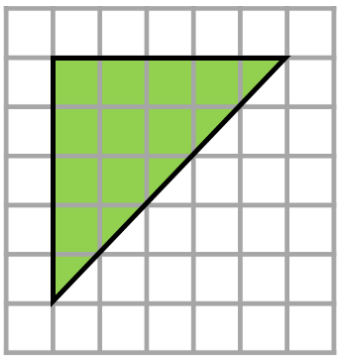


What do you notice about the area of the triangle compared to the area of the square?
Does this always happen?

Explore this using different rectangles.

Area of a Triangle (1)

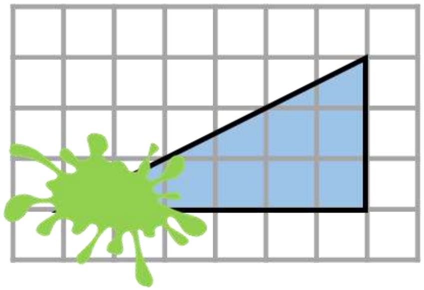
Reasoning and Problem Solving



Mo says the area of this triangle is 15cm²
Is Mo correct? If not, explain his mistake.

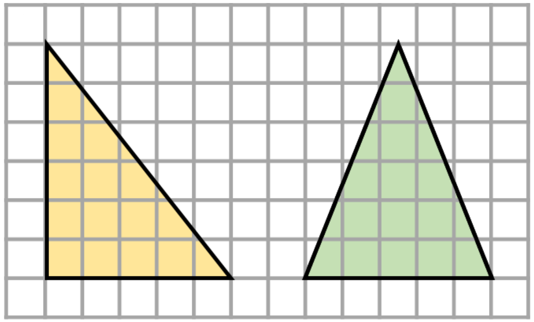
Mo is incorrect because he has counted the half squares as whole squares.

Part of a triangle has been covered.
Estimate the area of the whole triangle.



9 cm²

What is the same about these two triangles?
What is different?



Can you create a different right angled triangle with the same area?

Both triangles have an area of 15 cm²
The triangle on the left is a right angled triangle and the triangle on the right is an isosceles triangle.

Children could draw a triangle with a height of 10 cm and a base of 3 cm, or a height of 15 cm and a base of 2 cm.

Area of a Triangle (2)

Notes and Guidance

Children use their knowledge of finding the area of a rectangle to find the area of a right-angled triangle. They see that a right-angled triangle with the same length and perpendicular height as a rectangle will have an area half the size.
Using the link between the area of a rectangle and a triangle, children will learn and use the formula to calculate the area of a triangle.

Mathematical Talk

What is the same/different about the rectangle and triangle?

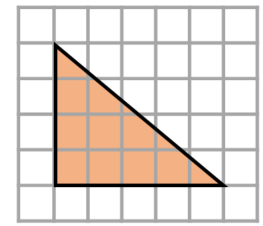
What is the relationship between the area of a rectangle and the area of a right-angled triangle?

What is the formula for working out the area of a rectangle or square?

How can you use this formula to work out the area of a right-angled triangle?

Varied Fluency

- Estimate the area of the triangle by counting the squares.
Make the triangle into a rectangle with the same height and width. Calculate the area.

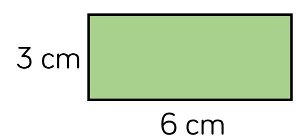


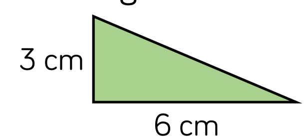
The area of the triangle is ________ the area of the rectangle.

- If l represents length and h represents height:

Area of a rectangle = $l \times h$

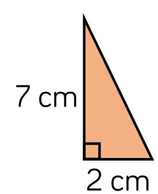
Use this to calculate the area of the rectangle.

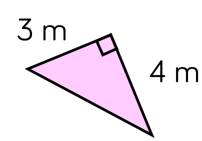


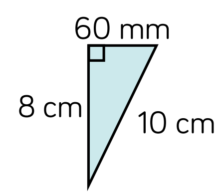


What do you need to do to your answer to work out the area of the triangle?
Therefore, what is the formula for the area of a triangle?

- Calculate the area of these triangles.



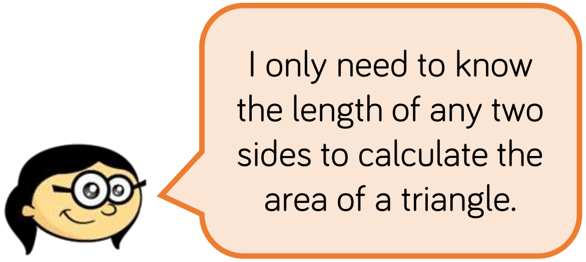




Area of a Triangle (2)

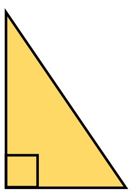
Reasoning and Problem Solving

Annie is calculating the area of a right-angled triangle.



Do you agree with Annie? Explain your answer.

Annie is incorrect as it is not sufficient to know **any** two sides, she needs the base and perpendicular height. Children could draw examples and non-examples.



Area = 54 cm²

What could the length and the height of the triangle be?

How many different integer possibilities can you find?

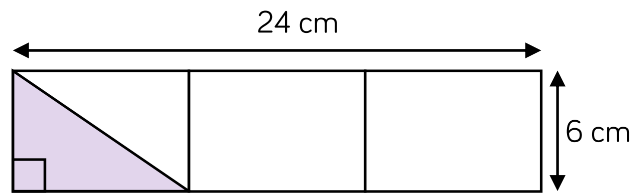
Possible answers:

Height: 18 cm
Base: 6 cm

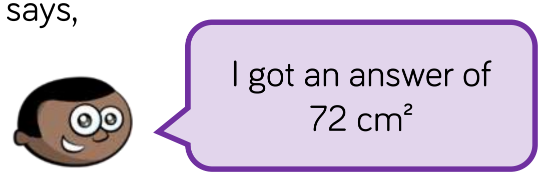
Height: 27 cm
Base: 4 cm

Height: 12 cm
Base: 9 cm

Calculate the area of the shaded triangle.



Mo says,



Do you agree with Mo?
If not, can you spot his mistake?

The area of the shaded triangle is 24 cm²

Mo is incorrect as he has just multiplied the two numbers given and divided by 2, he hasn't identified the correct base of the triangle.

Area of a Triangle (3)

Notes and Guidance

Children will extend their knowledge of working out the area of a right-angled triangle to work out the area of any triangle.

They use the formula, base × perpendicular height ÷ 2 to calculate the area of a variety of triangles where different side lengths are given and where more than one triangle make up a shape.

Mathematical Talk

What does the word perpendicular mean?

What do we mean by perpendicular height?

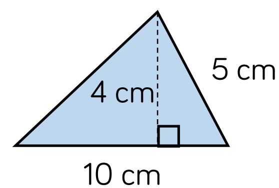
What formula can you use to calculate the area of a triangle?

If there is more than one triangle making up a shape, how can we use the formula to find the area of the whole shape?

How do we know which length tells us the perpendicular height of the triangle?

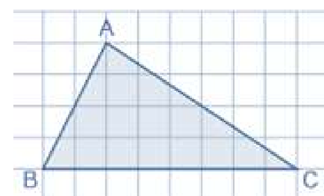
Varied Fluency

- To calculate the height of a triangle, you can use the formula:
  base × height ÷ 2
  Choose the correct calculation to find the area of the triangle.



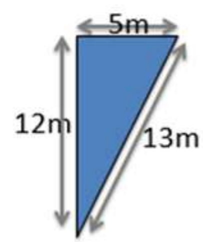
  - 10 × 5 ÷ 2
  - 10 × 4 ÷ 2
  - 5 × 4 ÷ 2

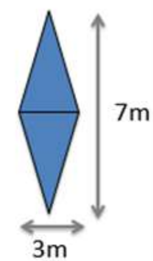
- Estimate the area of the triangle by counting squares.

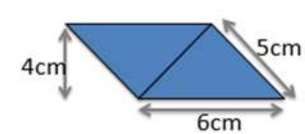


  Now calculate the area of the triangle.
  Compare your answers.

- Calculate the area of each shape.



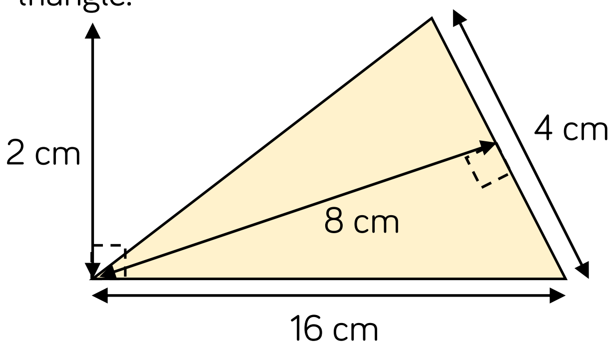




Area of a Triangle (3)

Reasoning and Problem Solving

Class 6 are calculating the area of this triangle.



Here are some of their methods.

$4 \times 8 \times 16 \times 2 \div 2$	$4 \times 8 \div 2$
$16 \times 2 \div 2$	$16 \times 4 \div 2$
$16 \times 8 \div 2$	8×1

Tick the correct methods.

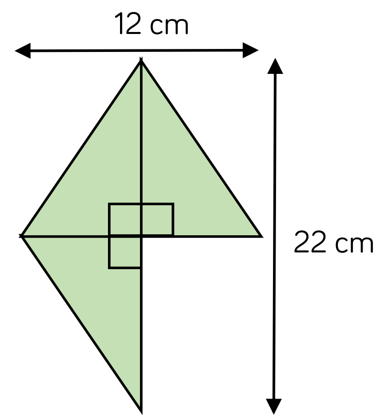
Explain any mistakes.

The correct methods are:
$16 \times 2 \div 2$
$4 \times 8 \div 2$

All mistakes are due to not choosing a pair of lengths that are perpendicular.

Children could explore other methods to get to the correct answer e.g. halving the base first and calculating 8×2 etc.

The shape is made of three identical triangles.



What is the area of the shape?

Each triangle is 6 cm by 11 cm so area of one triangle is 33 cm^2

Total area = 99 cm^2

Area of a Parallelogram

Notes and Guidance

Children use their knowledge of finding the area of a rectangle to find the area of a parallelogram.

Children investigate the link between the area of a rectangle and parallelogram by cutting a parallelogram so that it can be rearranged into a rectangle. This will help them understand why the formula to find the area of parallelograms works.

Mathematical Talk

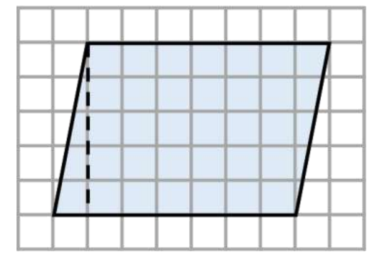
Describe a parallelogram.

What do you notice about the area of a rectangle and a parallelogram?

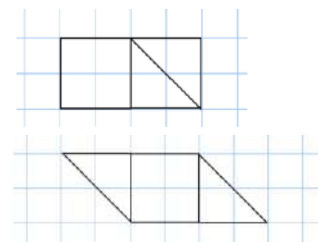
What formula can you use to work out the area of a parallelogram?

Varied Fluency

- Approximate the area of the parallelogram by counting squares.
Now cut along the dotted line.
Can you move the triangle to make a rectangle?
Calculate the area of the rectangle.

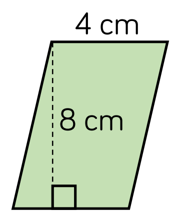


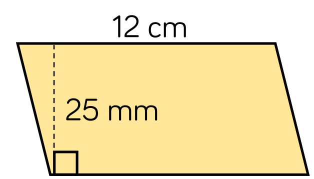
- Here are two quadrilaterals.

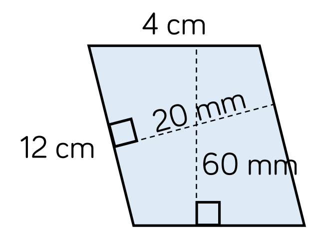


  - What is the same about the quadrilaterals?
  - What's different?
  - What is the area of each quadrilateral?

- Use the formula base × perpendicular height to calculate the area of the parallelograms.







Area of a Parallelogram

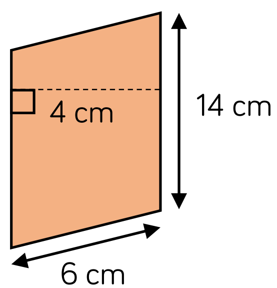
Reasoning and Problem Solving

Teddy has drawn a parallelogram.

The area is greater than 44 m^2 but less than 48 m^2.

What could the base length and the perpendicular height of Teddy's parallelogram be?

Possible answers:
9 m by 5 m
= 45 m^2
6.5 m by 7 m
= 45.5 m^2
11 m by 4.2 m
= 46.2 m^2



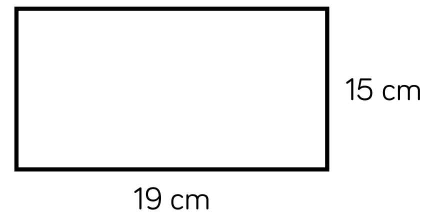
Dexter thinks the area of the parallelogram is 84 cm^2.

What mistake has Dexter made?

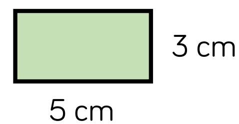
What is the correct area?

Dexter has multiplied 14 by 6 when he should have multiplied by 4 because 4 is the perpendicular height of the parallelogram.
The correct area is 56 cm^2.

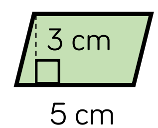
Dora and Eva are creating a mosaic.
They are filling a sheet of paper this size.



Dora is using tiles that are rectangular.



Eva's tiles are parallelograms.



Dora thinks that she will use fewer tiles than Eva to fill the page because her tiles are bigger.
Do you agree? Explain your answer.

Dora is wrong because both hers and Eva' s tiles have the same area and so the same number of tiles will be needed to complete the mosaic.
The area of the paper is 285 cm^2 and the area of each tile is 15 cm^2 so 19 tiles are needed to complete the pattern.

What is Volume?

Notes and Guidance

Children understand that volume is the amount of solid space something takes up. They look at how volume is different to capacity, as capacity is related to the amount a container can hold.
Children could use centimetre cubes to make solid shapes. Through this, they recognise the conservation of volume by building different solids using the same amount of centimetre cubes.

Mathematical Talk

Does your shape always have 4 centimetre cubes? Do they take up the same amount of space?
How can this help us understand what volume is?

If the solid shapes are made up of 1 cm cubes, can you complete the table?
Look at shape A, B and C. What's the same and what's different?

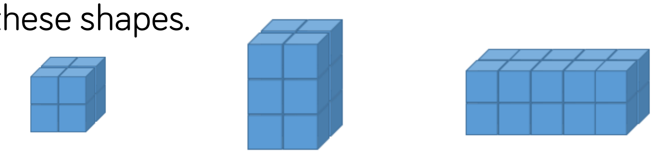
How is capacity different to volume?

Varied Fluency

R

- Take 4 cubes of length 1 cm. How many different solids can you make? What's the same? What's different?

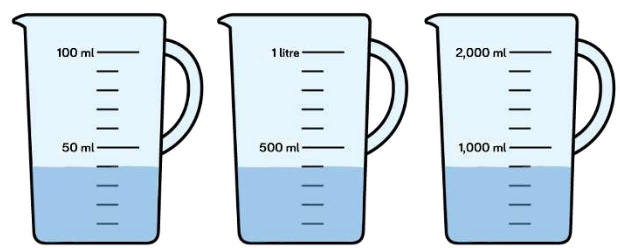
- Make these shapes.



Complete the table to describe your shapes.

Shape	Width (cm)	Height (cm)	Length (cm)	Volume (cm^3)
A				
B				
C				

- Compare the capacity and the volume. Use the sentence stems to help you.



Container ___ has a capacity of ____ ml
The volume of water in container ___ is ___ cm^3

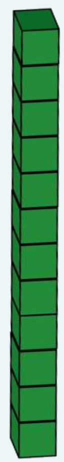
What is Volume?

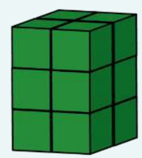
Reasoning and Problem Solving

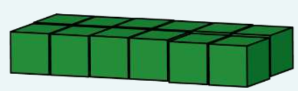
R

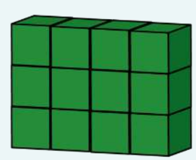
How many possible ways can you make a cuboid that has a volume of 12cm^3?

Possible solutions:









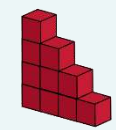
My shape is made up of 10 centimetre cubes.

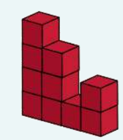
The height and length are the same size.

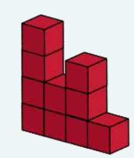
What could my shape look like?

Create your own shape and write some clues for a partner.

Possible solutions include:







Volume – Counting Cubes

Notes and Guidance

Children should understand that volume is the space occupied by a 3-D object.

Children will start by counting cubic units ($1\ cm^3$) to find the volume of 3D shapes. They will then use cubes to build their own models and describe the volume of the models they make.

Mathematical Talk

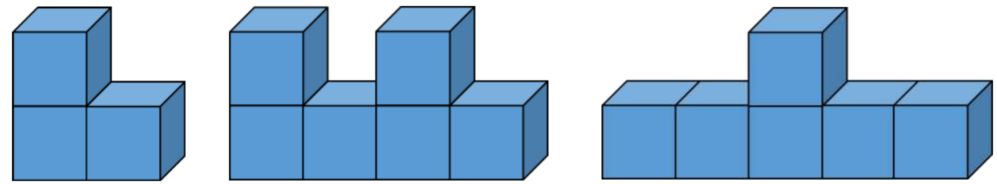
What's the same and what's different between area and volume?

Can you explain how you worked out the volume?
What did you visualise?

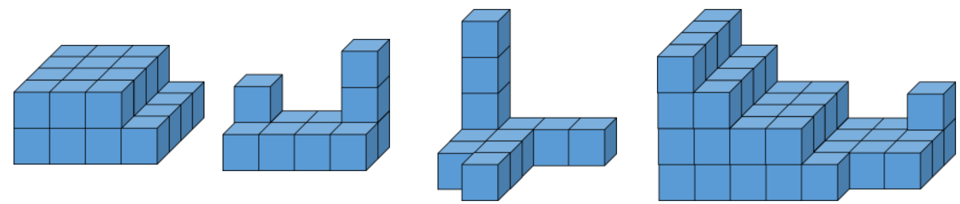
What units of measure could we use for volume? (Explore cm^3, m^3, mm^3 etc.)

Varied Fluency

- If each cube has a volume of $1\ cm^3$, find the volume of each solid.



- Make each shape using multilink cubes.



If each cube has a volume of $1\ cm^3$, what is the volume of each shape?
Place the shapes in ascending order based on their volume.
What about if each cube represented $1\ mm^3$, how would this affect your answer? What about if they were $1\ m^3$?

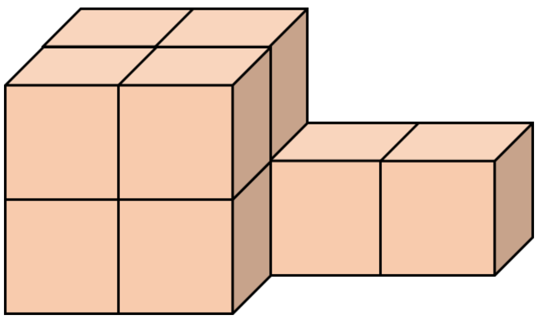
- If one multilink cube represents 1 cubic unit, how many different models can you make with a volume of 12 cubic units?

Volume – Counting Cubes

Reasoning and Problem Solving

Amir says he will need 8 cm^3 to build this shape.

Dora says she will need 10 cm^3.



Who do you agree with?

Explain why.

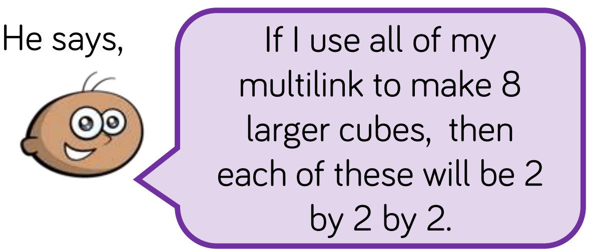
Amir is incorrect because he has missed the 2 cubes that cannot be seen.

Dora is correct because there are 8 cm^3 making the visible shape, then there are an additional 2 cm^3 behind.

Tommy is making cubes using multilink. He has 64 multilink cubes altogether.

How many different sized cubes could he make?

He says,



How many other combinations can Tommy make where he uses all the cubes?

Tommy could make:

- $1 \times 1 \times 1$
- $2 \times 2 \times 2$
- $3 \times 3 \times 3$
- $4 \times 4 \times 4$

Possible answers:

64 cubes that are $1 \times 1 \times 1$

2 cubes that are $3 \times 3 \times 3$; 1 cube that is $2 \times 2 \times 2$; 2 cubes that are $1 \times 1 \times 1$

White Rose Maths

Volume of a Cuboid

Notes and Guidance

Children make the link between counting cubes and the formula ($l \times w \times h$) for calculating the volume of cuboids.

They realise that the formula is the same as calculating the area of the base and multiplying this by the height.

Mathematical Talk

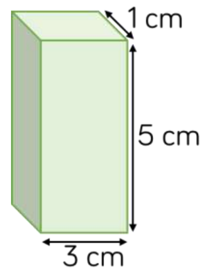
Can you identify the length, width and height of the cuboid?

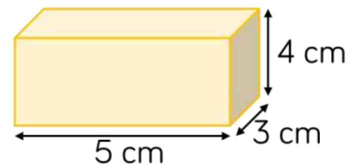
If the length of a cuboid is 5 cm and the volume is 100 cm^3, what could the width and height of the cuboid be?

What knowledge can I use to help me calculate the missing lengths?

Varied Fluency

- Complete the sentences for each cuboid.





The length is: ________
The width is: ________
The height is: ________

The area of the base is: ____ × ____ = ____
Volume = The area of the base × ____ = ____

- Calculate the volume of a cube with side length:

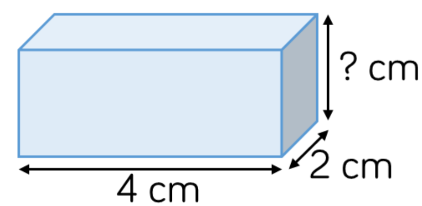
4 cm   2 m   160 mm

Use appropriate units for your answers.

- The volume of the cuboid is 32 cm^3.

Calculate the height.

You might want to use multilink cubes to help you.

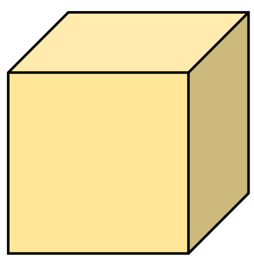


Volume of a Cuboid

Reasoning and Problem Solving

Rosie says,

You can't calculate the volume of the cube because you don't know the width or the height.



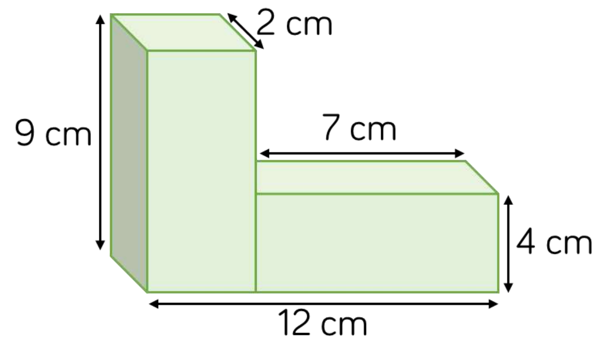
Do you agree?

Explain why.

You don't need the rest of the measurements because it's a cube and all the edges of a cube are equal.
Therefore, the width would be 2 cm and the height would be 2 cm.

The volume of the cube is 8 cm³

Calculate the volume of the shape.



146 cm³

How many different ways can you make a cuboid with a volume of 48 cm³?

Possible answers:

24 × 2 × 1
2 × 6 × 4
6 × 8 × 1

White Rose Maths

Spring - Block 6

Ratio

Overview

Small Steps

- Using ratio language
- Ratio and fractions
- Introducing the ratio symbol
- Calculating ratio
- Using scale factors
- Calculating scale factors
- Ratio and proportion problems

Notes for 2020/21

All of this block is new learning for Year 6 so there are no recap steps.

Bar models are a key representation in this topic. Children may need some extra input here if they have not used bar models throughout KS2.

Using Ratio Language

Notes and Guidance

Children will understand that a ratio shows the relationship between two values and can describe how one is related to another.

They will start by making simple comparisons between two different quantities. For example, they may compare the number of boys to girls in the class and write statements such as, "For every one girl, there are two boys".

Mathematical Talk

How would your sentences change if there were 2 more blue flowers?

How would your sentences change if there were 10 more pink flowers?

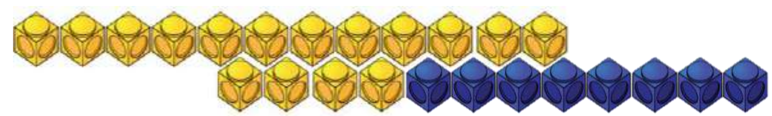
Can you write a "For every…" sentence for the number of boys and girls in your class?

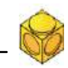
Varied Fluency

- Complete the sentences.

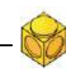
For every two blue flowers there are ____ pink flowers.
For every blue flower there are _____ pink flowers.

- Use cubes to help you complete the sentences.

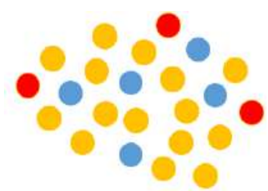


For every ____ , there are ____ 

For every 8 , there are ____ 

For every 1 , there are ____ 

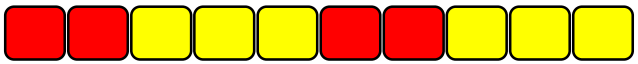
- How many "For every…" sentences can you write to describe these counters?



Using Ratio Language

Reasoning and Problem Solving

Whitney lays tiles in the following pattern



If she has 16 red tiles and 20 yellow tiles remaining, can she continue her pattern without there being any tiles left over?

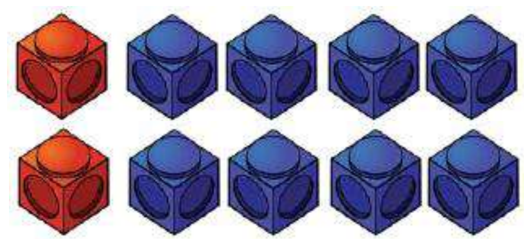
Explain why.

Possible responses:

For every two red tiles there are three yellow tiles. If Whitney continues the pattern she will need 16 red tiles and 24 yellow tiles. She cannot continue the pattern without there being tiles left over.

20 is not a multiple of 3

True or False?



For every red cube there are 8 blue cubes.	False
For every 4 blue cubes there is 1 red cube.	True
For every 3 red cubes there would be 12 blue cubes.	True
For every 16 cubes, 4 would be red and 12 would be blue.	False
For every 20 cubes, 4 would be red and 16 would be blue.	True

Ratio and Fractions

Notes and Guidance

Children often think a ratio 1 : 2 is the same as a fraction of $\frac{1}{2}$

In this step, they use objects and diagrams to compare ratios and fractions.

Mathematical Talk

How many counters are there altogether?

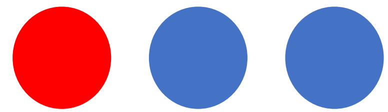
How does this help you work out the fraction?

What does the denominator of the fraction tell you?

How can a bar model help you to show the mints and chocolates?

Varied Fluency

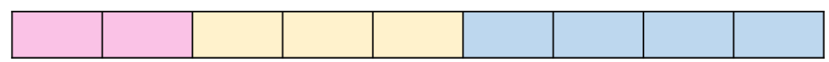
- The ratio of red counters to blue counters is 1 : 2



What fraction of the counters is blue? $\frac{1}{2}$ $\frac{1}{3}$ $\frac{2}{3}$

What fraction of the counters is red? $\frac{1}{2}$ $\frac{1}{3}$ $\frac{2}{3}$

- This bar model shows the ratio 2 : 3 : 4



What fraction of the bar is pink?
What fraction of the bar is yellow?
What fraction of the bar is blue?

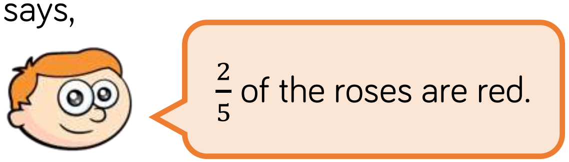
- One third of the sweets in a box are mints.
The rest are chocolates.
What is the ratio of mints to chocolates in the box?

Ratio and Fractions

Reasoning and Problem Solving

Ron plants flowers in a flower bed.
For every 2 red roses he plants 5 white roses.

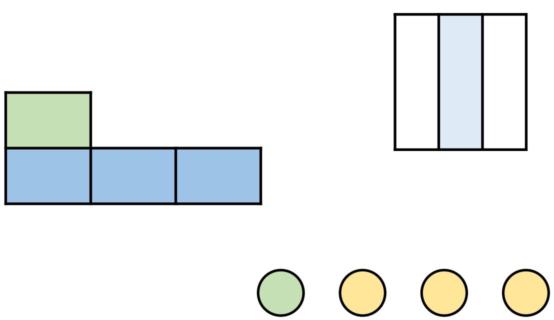
He says,

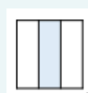


Is Ron correct?

Ron is incorrect because $\frac{2}{7}$ of the roses are red. He has mistaken a part with the whole.

Which is the odd one out?
Explain your answer.



 is the odd one out because one part out of three is a different colour. The others are one part out of four.

There are some red and green cubes in a bag. $\frac{2}{5}$ of the cubes are red.

True or False?

• For every 2 red cubes there are 5 green cubes.	False
• For every 2 red cubes there are 3 green cubes.	True
• For every 3 green cubes there are 2 red cubes.	True
• For every 3 green cubes there are 5 red cubes.	False

Explain your answers.

Introducing the Ratio Symbol

Notes and Guidance

Children are introduced to the colon notation as the ratio symbol, and continue to link this with the language 'for every…, there are…'
They need to read ratios e.g. 3 : 5 as "three to five".
Children understand that the notation relates to the order of parts. For example, 'For every 3 bananas there are 2 apples would be the same as 3 : 2 and for every 2 apples there are 3 bananas would be the same as 2 : 3

Mathematical Talk

What does the : symbol mean in the context of ratio?

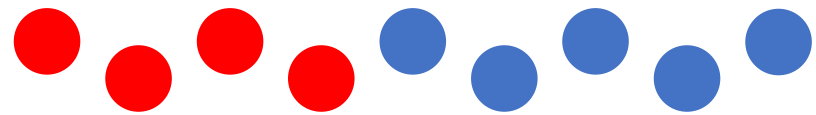
Why is the order of the numbers important when we write ratios?

How do we write a ratio that compares three quantities?

How do we say the ratio "3 : 7"?

Varied Fluency

Complete:



The ratio of red counters to blue counters is ☐ : ☐

The ratio of blue counters to red counters is ☐ : ☐

Write down the ratio of:

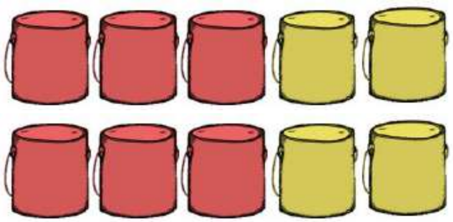
- Bananas to strawberries
- Blackberries to strawberries
- Strawberries to bananas to blackberries
- Blackberries to strawberries to bananas

The ratio of red to green marbles is 3 : 7
Draw an image to represent the marbles.
What fraction of the marbles are red?
What fraction of the marbles are green?

Introducing the Ratio Symbol

Reasoning and Problem Solving

Tick the correct statements.



- There are two yellow tins for every three red tins.
- There are two red tins for every three yellow tins.
- The ratio of red tins to yellow tins is 2 : 3
- The ratio of yellow tins to red tins is 2 : 3

Explain which statements are incorrect and why.

The first and last statement are correct. The other statements have the ratios the wrong way round.

In a box there are some red, blue and green pens.

The ratio of red pens to green pens is 3 : 5

For every 1 red pen there are two blue pens.

Write down the ratio of red pens to blue pens to green pens.

R : G

3 : 5

R : B

1 : 2 or

3 : 6

R : B : G

3 : 6 : 5

Calculating Ratio

Notes and Guidance

Children build on their knowledge of ratios and begin to calculate ratios. They answer worded questions in the form of 'for every… there are …' and need to be able to find both a part and a whole.
They should be encouraged to draw bar models to represent their problems, and clearly label the information they have been given and what they want to calculate.

Mathematical Talk

How can we represent this ratio using a bar model?

What does each part represent? What will each part be worth?

How many parts are there altogether? What is each part worth?

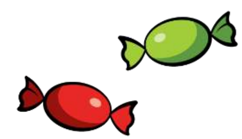
If we know what one part is worth, can we calculate the other parts?

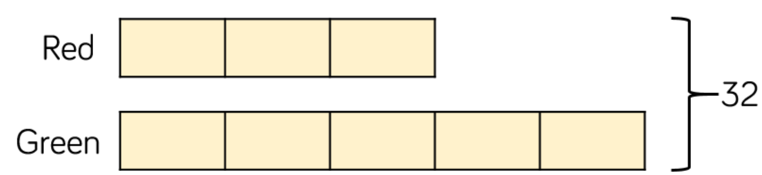
Varied Fluency

- A farmer plants some crops in a field.
For every 4 carrots he plants 2 leeks.
He plants 48 carrots in total.
How many leeks did he plant?
How many vegetables did he plant in total?

- Jack mixes 2 parts of red paint with 3 parts blue paint to make purple paint.
If he uses 12 parts blue paint, how many parts red paint does he use?

- Eva has a packet of sweets.
For every 3 red sweets there are 5 green sweets.
If there are 32 sweets in the packet in total, how many of each colour are there?
You can use a bar model to help you.





Calculating Ratio

Reasoning and Problem Solving

Teddy has two packets of sweets.

In the first packet, for every one strawberry sweet there are two orange sweets.

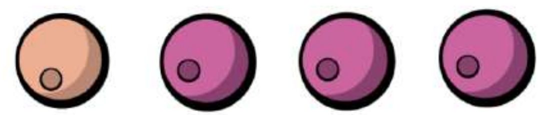
In the second packet, for every three orange sweets there are two strawberry sweets.

Each packet contains 15 sweets in total.

Which packet has more strawberry sweets and by how many?

The first packet has 5 strawberry sweets and 10 orange sweets.
The second packet has 6 strawberry sweets and 9 orange sweets.
The second packet has 1 more strawberry sweet than the first packet.

Annie is making some necklaces to sell. For every one pink bead, she uses three purple beads.



Each necklace has 32 beads in total.

The cost of the string is £2.80
The cost of a pink bead is 72p.
The cost of a purple bead is 65p.

How much does it cost to make one necklace?

Each necklace has 8 pink beads and 24 purple beads.

The cost of the pink beads is £5.76

The cost of the purple beads is £15.60

The cost of a necklace is £24.16

Using Scale Factors

Notes and Guidance

In this step, children enlarge shapes to make them 2 or 3 times as big etc. They need to be introduced to the term "scale factor" as the name for this process.

Children should be able to draw 2-D shapes on a grid to a given scale factor and be able to use vocabulary, such as, "Shape A is three times as big as shape B".

Mathematical Talk

What does enlargement mean?

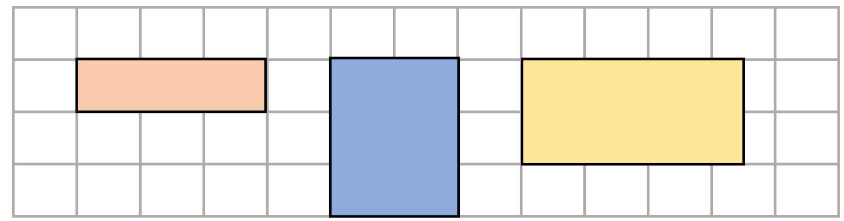
What does scale factor mean?

Why do we have to double/triple all the sides of each shape?

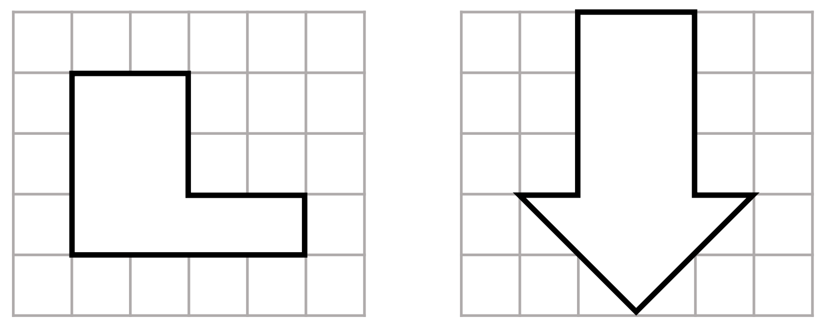
Have the angles changed size?

Varied Fluency

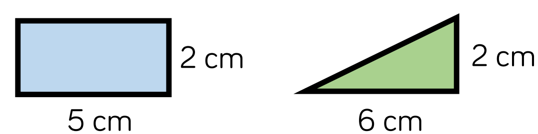
- Copy these rectangles onto squared paper then draw them double the size, triple the size and 5 times as big.



- Copy these shapes onto squared paper then draw them twice as big and three times as big.



- Enlarge these shapes by:
  - Scale factor 2
  - Scale factor 3
  - Scale factor 4



Using Scale Factors

Reasoning and Problem Solving

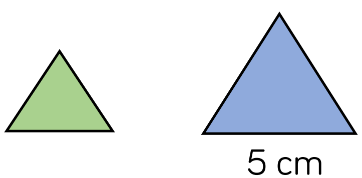
Draw a rectangle 3 cm by 4 cm.

Enlarge your rectangle by scale factor 2.

Compare the perimeter, area and angles of your two rectangles.

The perimeter has doubled, the area is four times as large, the angles have stayed the same.

Here are two equilateral triangles.
The blue triangle is three times larger than the green triangle.



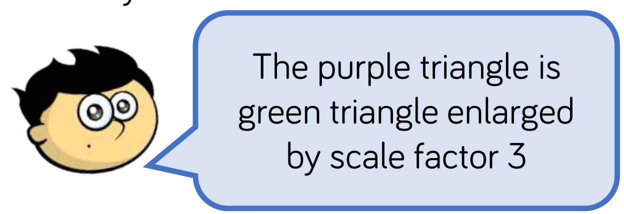
(Not drawn to scale)

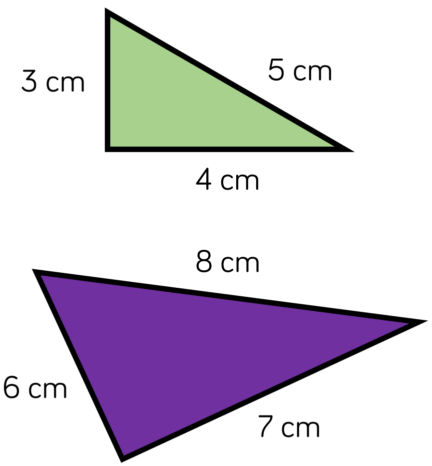
Find the perimeter of both triangles.

The blue triangle has a perimeter of 15 cm.

The green triangle has a perimeter of 5 cm.

Jack says:





Do you agree?
Explain why.

Possible answer
I do not agree because Jack has increased the green shape by adding 3 cm to each side, not increasing it by a scale factor of 3

Calculating Scale Factors

Notes and Guidance

Children find scale factors when given similar shapes. They need to be taught that 'similar' in mathematics means that one shape is an exact enlargement of the other, not just they have some common properties.

Children use multiplication and division facts to calculate missing information and scale factors.

Mathematical Talk

What does similar mean?

What do you notice about the length/width of each shape?

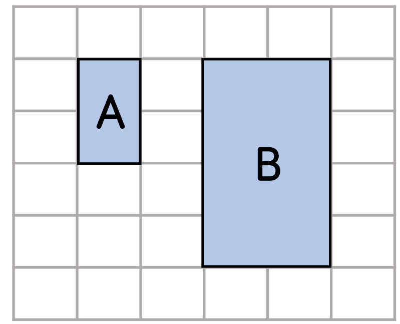
How would drawing the rectangles help you?

How much larger/smaller is shape A compared to shape B?

What does a scale factor of 2 mean? Can you have a scale factor of 2.5?

Varied Fluency

Complete the sentences.



Shape B is __________ as big as shape A.

Shape A has been enlarged by scale factor _____ to make shape B.

The rectangles described in the table are all similar to each other. Fill in the missing lengths and widths and complete the sentences.

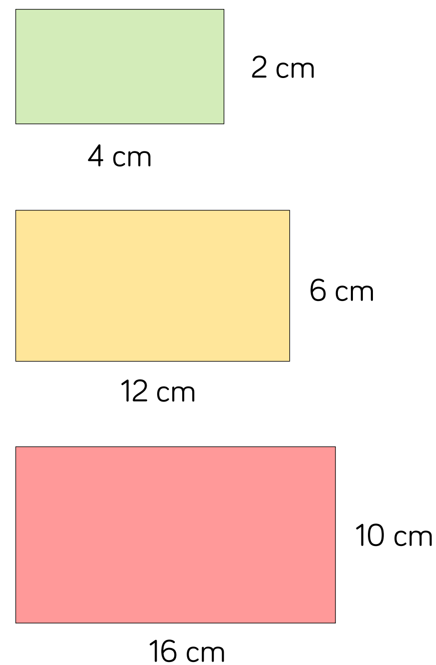
Rectangle	Length	Width
A	5 cm	2 cm
B		4 cm
C	25 cm	
D		18 cm

From A to B, the scale factor of enlargement is ____
From A to C, the scale factor of enlargement is ____
From A to D the scale factor of enlargement is ____
From B to D, the scale factor of enlargement is ____

Calculating Scale Factors

Reasoning and Problem Solving

A rectangle has a perimeter of 16 cm. An enlargement of this rectangle has a perimeter of 24 cm. The length of the smaller rectangle is 6 cm. Draw both rectangles.	Smaller rectangle: length – 6 cm width – 2 cm Larger rectangle: length – 9 cm width – 3 cm Scale factor: 1.5
Always, sometimes, or never true? To enlarge a shape you just need to do the same thing to each of the sides.	Sometimes. This only works when we are multiplying or dividing the lengths of the sides. It does not work when adding or subtracting.

Ron says that these three rectangles are similar.  Do you agree? Explain your answer.	Ron is incorrect. The orange rectangle is an enlargement of the green rectangle with scale factor 3. The red rectangle, however, is not similar to the other two as the side lengths are not in the same ratio.

Ratio and Proportion Problems

Notes and Guidance

Children will apply the skills they have learnt in the previous steps to a wide range of problems in different contexts.

They may need support to see that different situations are in fact alternative uses of ratio.

Bar models will again provide valuable pictorial support.

Mathematical Talk

How does this problem relate to ratio?

Can we represent this ratio using a bar model?

What does each part represent? What is the whole?

What is the same about the ratios?

What is different about them?

Varied Fluency

How much of each ingredient is needed to make soup for:
- 3 people
- 9 people
- 1 person

What else could you work out?

Recipe for 6 people
- 1 onion
- 60 g butter
- 180 g lentils
- 1.2 litres stock
- 480 ml tomato juice

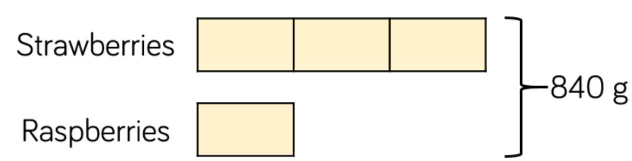
Two shops sell the same pens for these prices.

Safeway
4 pens £2.88

K-mart
7 pens £4.83

Which shop is better value for money?

The mass of strawberries in a smoothie is three times the mass of raspberries in the smoothie. The total mass of the fruit is 840 g. How much of each fruit is needed.



Ratio and Proportion Problems

Reasoning and Problem Solving

This recipe makes 10 flapjacks.

Flapjacks

120 g butter
100 g brown sugar
4 tablespoons golden syrup
250 g oats
40 g sultanas

Amir has 180 g butter.

What is the largest number of flapjacks he can make?

How much of the other ingredients will he need?

He has enough butter to make 15 flapjacks.
He will need 150 g brown soft sugar, 6 tablespoons golden syrup, 375 g oats and 60 g sultanas.

Alex has two packets of sweets.

In the first packet, for every 2 strawberry sweets there are 3 orange.

In the second packet, for one strawberry sweet, there are three orange.

Each packet has the same number of sweets.

The second packet contains 15 orange sweets.

How many strawberry sweets are in the first packet?

Second packet:
15 orange
5 strawberry.

So there are 20 sweets in each packet.

First packet:
8 strawberry
12 orange

The first packet contains 8 strawberry sweets.

Spring - Block 7

Statistics

Overview

Small Steps

- Read and interpret line graphs
- Draw line graphs
- Use line graphs to solve problems
- Circles
- Read and interpret pie charts
- Pie charts with percentages
- Draw pie charts
- The mean

Notes for 2020/21

Time is limited at this stage in Year 6. Line graphs have been covered extensively in Year 4 and 5 so you may choose to skip these steps or merge them into one lesson. This will leave more time for pie charts and the mean.

Read and Interpret Line Graphs

Notes and Guidance

Children will build on their experience of interpreting data in context from Year 5, using their knowledge of scales to read information accurately. Examples of graphs are given but it would be useful if real data from across the curriculum e.g. Science, was also used. Please note that line graphs represent continuous data not discrete data. Children need to read information accurately, including where more than one set of data is on the same graph.

Mathematical Talk

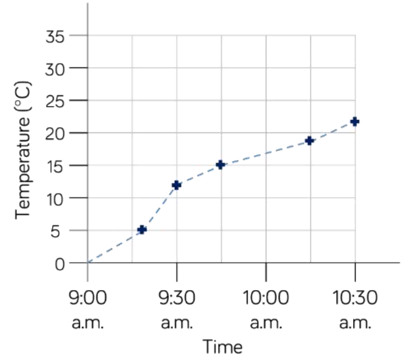
Where might you see a line graph used in real life?

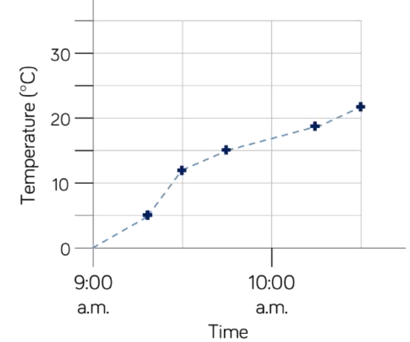
Why is the 'Water Consumption' graph more difficult to interpret?

How can you make sure that you read the information accurately?

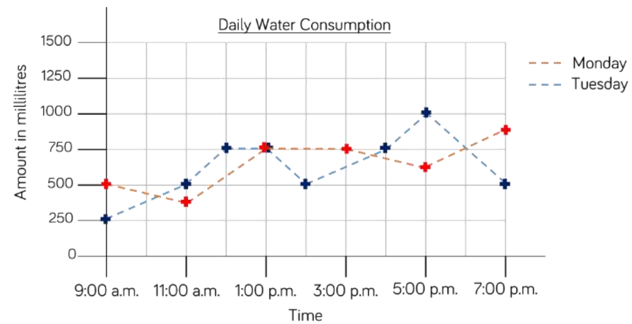
Varied Fluency

- What is the same and what is different about the two graphs?





- Here is a graph showing daily water consumption over two days.

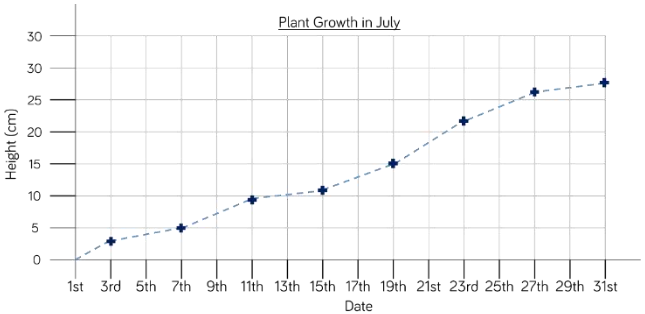


At what times of the day was the same amount of water consumed on Monday and Tuesday?
Was more water consumed at 2 p.m. on Monday or Tuesday morning? How much more?

Read and Interpret Line Graphs

Reasoning and Problem Solving

Eva has created a graph to track the growth of a plant in her house.



Eva recorded the following facts about the graph.

a) On the 9th of July the plant was about 9 cm tall.

b) Between the 11th and 19th July the plant grew about 5 cm.

c) At the end of the month the plant was twice as tall as it had been on the 13th .

Can you spot and correct Eva's mistakes?

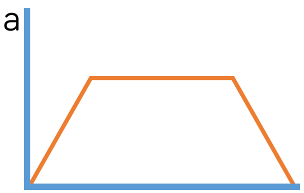
a) On the 9th July a more accurate measurement would be 7.5 cm.

b) Correct.

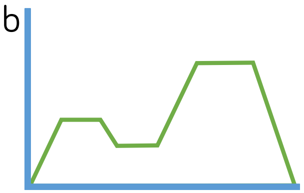
c) On the 31st the plant was approximately 28 cm tall, but on the 13th it was only 10 cm which is not half of 28 cm. The plant was closer to 14 cm on the 17th July.

Write a story and 3 questions for each of the 3 graphs below.

a



b



c

Possible context for each story:

a) A car speeding up, travelling at a constant speed, then slowing down.

b) The height above sea level a person is at during a walk.

c) Temperature in an oven when you are cooking something.

Draw Line Graphs

Notes and Guidance

Children will build on their experience of reading and interpreting data in order to draw their own line graphs.

Although example contexts are given, it would be useful if children can see real data from across the curriculum.

Children will need to decide on the most appropriate scales and intervals to use depending on the data they are representing.

Mathematical Talk

What will the x-axis represent? What intervals will you use?

What will the y-axis represent? What intervals will you use?

How will you make it clear which line represents which set of data?

Why is it useful to have both sets of data on one graph?

Varied Fluency

This table shows the height a rocket reached between 0 and 60 seconds.

Create a line graph to represent the information.

Time (seconds)	Height (metres)
0	0
10	8
20	15
30	25
40	37
50	50
60	70

The table below shows the population in the UK and Australia from 1990 to 2015.

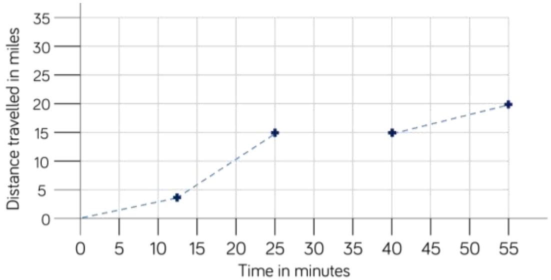
	1990	1995	2000
UK	57,200,000	58,000,000	58,900,000
Australia	17,000,000	18,000,000	19,000,000
	2005	2010	2015
UK	60,300,000	63,300,000	65,400,000
Australia	20,200,000	22,100,000	23,800,000

Create one line graph to represent the population in both countries. Create three questions to ask your friend about your completed graph.

Draw Line Graphs

Reasoning and Problem Solving

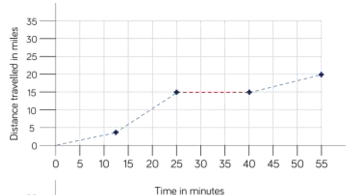
This graph shows the distance a car travelled.



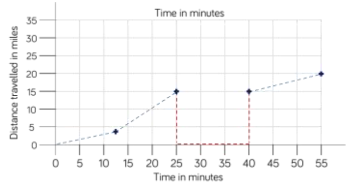
Rosie and Jack were asked to complete the graph to show the car had stopped.

Here are their completed graphs.

Rosie:



Jack:



Who has completed the graph correctly? Explain how you know.

Rosie has completed the graph correctly. The car has still travelled 15 miles in total, then stopped for 15 minutes before carrying on.

This table shows the distance a lorry travelled during the day.

Time	Distance in miles
7.00 a.m.	10
8.00 a.m.	28
9.00 a.m.	42
10.00 a.m.	58
11.00 a.m.	70
12.00 a.m.	95
1.00 p.m.	95
2.00 p.m.	118

Create a line graph to represent the information, where the divisions along the x-axis are every two hours.

Create a second line graph where the divisions along the x-axis are every hour.

Compare your graphs. Which graph is more accurate?

Would a graph with divisions at each half hour be even more accurate?

Children may find that the second line graph is easier to draw and interpret as it matches the data given directly.

They may discuss that it would be difficult to draw a line graph showing half hour intervals, as we cannot be sure the distance travelled at each half hour.

Line Graphs Problems

Notes and Guidance

Once children can read, interpret and draw lines graphs they need to be able to use line graphs to solve problems.

Children need to use their knowledge of scales to read information accurately. They need to be exposed to graphs that show more than one set of data.

At this point, children should be secure with the terms x and y axis, frequency and data.

Mathematical Talk

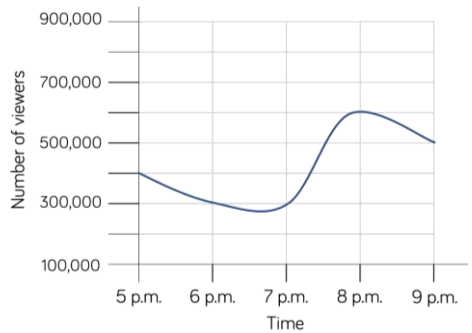
What do you notice about the scale on the vertical axis? Why might it be misleading?
What other scale could you use?

How is the information organised? Is it clear?
What else does this graph tell you? What does it not tell you?

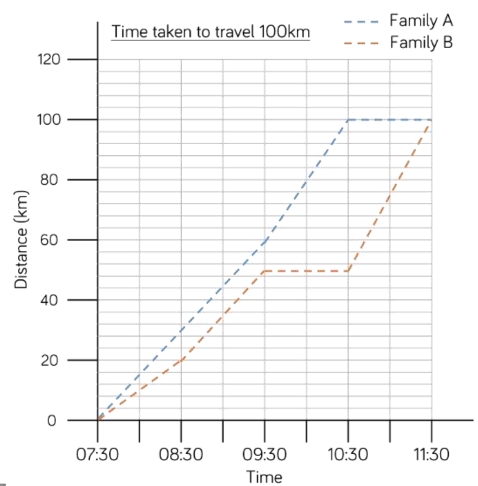
How can you calculate ________?
Why would this information be placed on a line graph and not a different type of graph?

Varied Fluency

Ron and Annie watched the same channel, but at different times.
The graph shows the number of viewers at different times.
Ron watched 'Chums' at 5 p.m. Annie watched 'Countup' at 8 p.m.



What was the difference between the number of viewers at the start of each programme? What was the difference in the number of viewers between 6 p.m. and 8 p.m.? Which time had twice as many viewers as 6 p.m.?

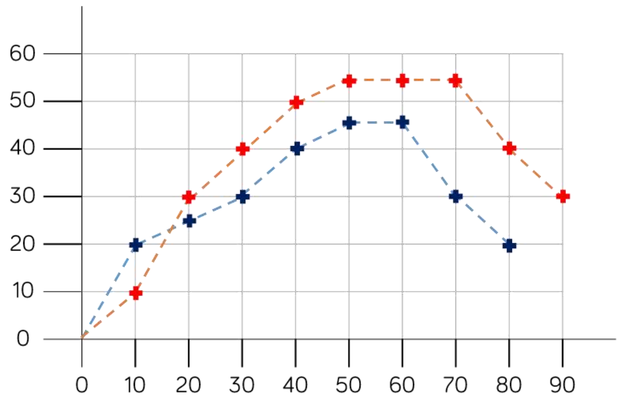


Two families were travelling to Bridlington for their holidays. They set off at the same time but arrived at different times.
What time did family A arrive?
How many km had each family travelled at 08:45?
Which family stopped midway through their journey?
How much further had they left to travel?

Line Graphs Problems

Reasoning and Problem Solving

What could this graph be showing?



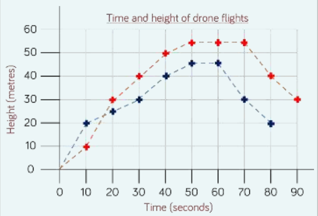
Label the horizontal and vertical axes to show this.

Is there more than one way to label the axes?

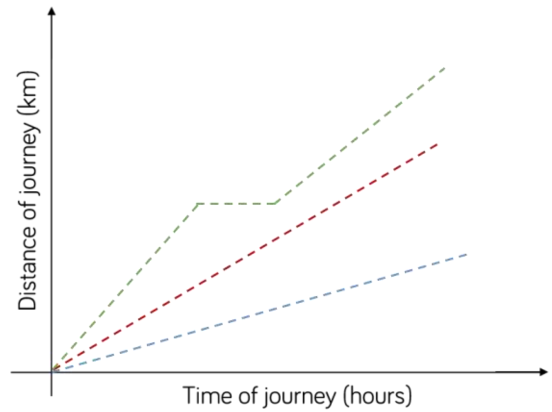
Possible response:

This graph shows the height of two drones and the time they were in the air.

For example:



The graph below shows some of Mr Woolley's journeys.



What is the same and what is different about each of these journeys?

What might have happened during the green journey?

Possible responses:

All the journeys were nearly the same length of time.

The journeys were all different distances.

The red and blue journey were travelling at constant speeds but red was travelling quicker than blue.

During the green journey, Mr Woolley might have been stuck in traffic or have stopped for a rest.

Circles

Notes and Guidance

Children will illustrate and name parts of circles, using the words radius, diameter, centre and circumference confidently.

They will also explore the relationship between the radius and the diameter and recognise the diameter is twice the length of the radius.

Mathematical Talk

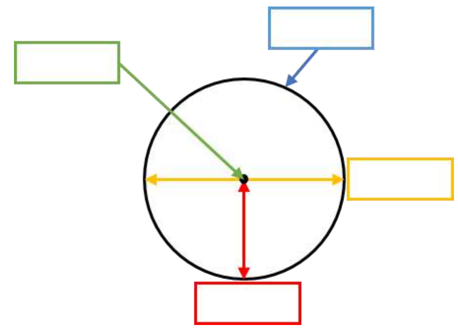
Why is the centre important?

What is the relationship between the diameter and the radius? If you know one of these, how can you calculate the other?

Can you use the vocabulary of a circle to describe and compare objects in the classroom?

Varied Fluency

- Using the labels complete the diagram:



Radius

Diameter

Centre

Circumference

- Find the radius or the diameter for each object below:

The radius is ___. The diameter is ___. I know this because ___.

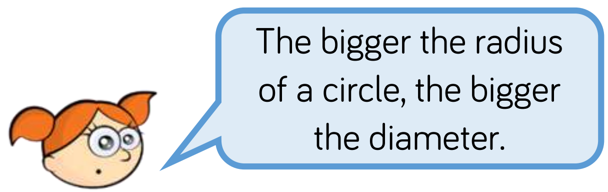
- Complete the table:

Radius	Diameter
26 cm	
	37 mm
2.55 m	
	99 cm
	19.36 cm

Circles

Reasoning and Problem Solving

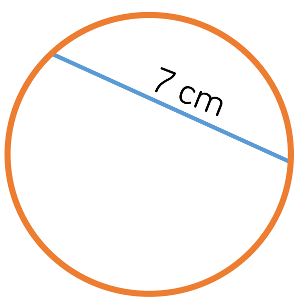
Alex says:



Do you agree? Explain your reasoning.

I agree with Alex because the diameter is always twice the length of the radius.

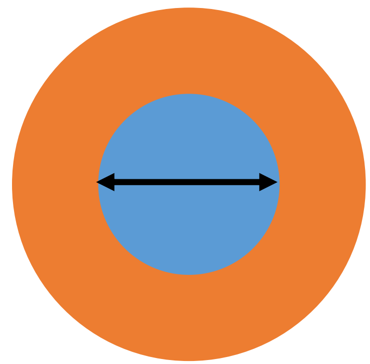
Spot the mistake!
Tommy has measured and labelled the diameter of the circle below.
He thinks that the radius of this circle will be 3.5 cm.



Is Tommy right? Explain why.

Tommy has measured the diameter inaccurately because the diameter always goes through the centre of the circle from one point on the circumference to another.

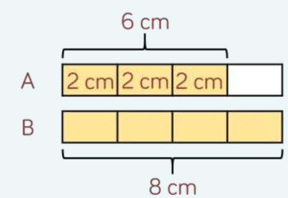
Here are 2 circles. Circle A is blue; Circle B is orange. The diameter of Circle A is $\frac{3}{4}$ the diameter of Circle B.



If the diameter of Circle B is 12 cm, what is the diameter of Circle A?
If the diameter of Circle A is 12 cm, what is the radius of Circle B?
If the diameter of Circle B is 6 cm, what is the diameter of Circle A?
If the diameter of Circle A is 6 cm, what is the radius of Circle B?

a) 9 cm
b) 16 cm
c) 4.5 cm
d) 8 cm

A bar model may support children in working these out e.g.



Read and Interpret Pie Charts

Notes and Guidance

Children will build on their understanding of circles to start interpreting pie charts. They will understand how to calculate fractions of amounts to interpret simple pie charts.

Children should understand what the whole of the pie chart represents and use this when solving problems.

Mathematical Talk

What does the whole pie chart represent? What does each colour represent?

Do you recognise any of the fractions? How can you use this to help you?

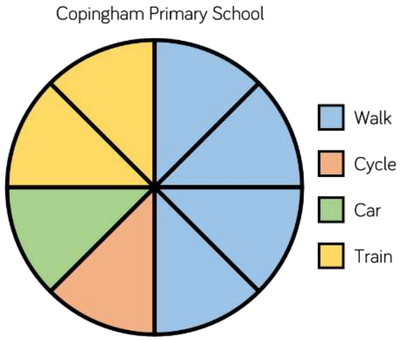
What's the same and what's different about the favourite drinks pie charts?

What other questions could you ask about the pie chart?

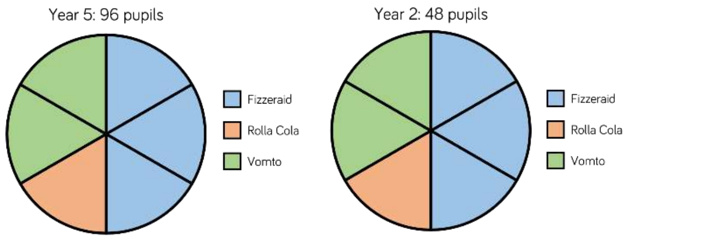
Varied Fluency

- There are 600 pupils at Copingham Primary school. Work out how many pupils travel to school by:

  a) Train
  b) Car
  c) Cycling
  d) Walking



- Classes in Year 2 and Year 5 were asked what their favourite drink was. Here are the results:



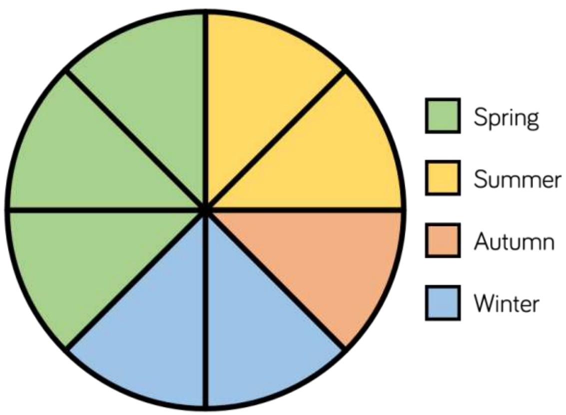
What fraction of pupils in Year 5 chose Fizzeraid?
How many children in Year 2 chose Rolla Cola?
How many more children chose Vomto than Rolla Cola in Year 2?
What other questions could you ask?

Read and Interpret Pie Charts

Reasoning and Problem Solving

In a survey people were asked what their favourite season of the year was. The results are shown in the pie chart below. If 48 people voted summer, how many people took part in the survey?

Our favourite time of year

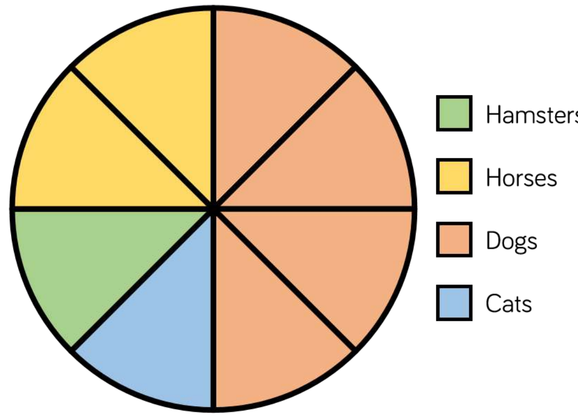


Explain your method.

Summer is a quarter of the whole pie chart and there are 4 quarters in a whole, so $48 \times 4 = 184$ people in total.

96 people took part in this survey.

Our favourite pets



How many people voted for cats?

$\frac{3}{8}$ of the people who voted for dogs were male. How many females voted for dogs?

What other information can you gather from the pie chart?

Write some questions about the pie chart for your partner to solve.

$\frac{1}{2}$ of $96 = 48$

$\frac{1}{4}$ of $96 = 24$

$\frac{1}{8}$ of $96 = 12$

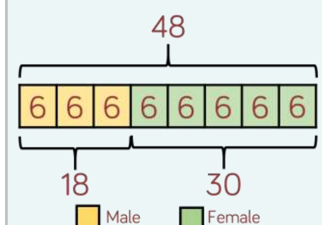
12 people voted cats.

48 people voted dogs.

$\frac{1}{8}$ of $48 = 6$

$6 \times 3 = 18.$

18 females voted for dogs.



Pie Charts With Percentages

Notes and Guidance

Children will apply their understanding of calculating percentages of amounts to interpret pie charts.

Children know that the whole of the pie chart totals 100 %.

Encourage children to recognise fractions in order to read the pie chart more efficiently.

Mathematical Talk

How did you calculate the percentage? What fraction knowledge did you use?

How else could you find the difference between Chocolate and Mint Chocolate?

If you know 5 % of a number, how can you work out the whole number?

If you know what 5 % is, what else do you know?

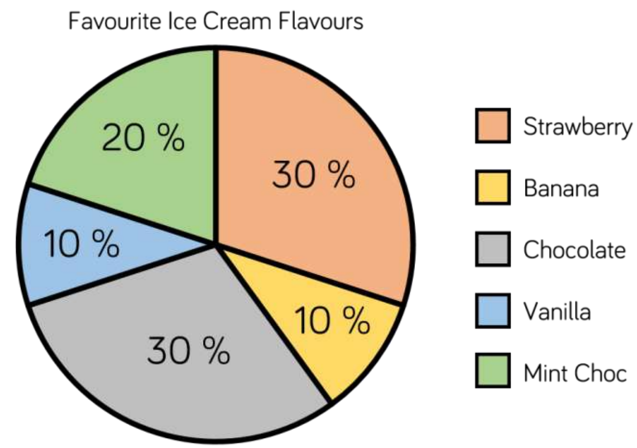
Varied Fluency

- 150 children voted for their favourite ice cream flavours. Here are their results:

  How many people voted for Vanilla?

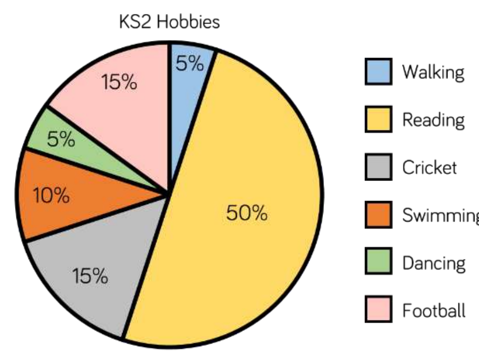
  How many more people voted for Chocolate than Mint Chocolate Chip?

  How many people chose Chocolate, Banana and Vanilla altogether?



- There are 200 pupils in Key Stage 2 who chose their favourite hobbies.

  How many pupils chose each hobby?

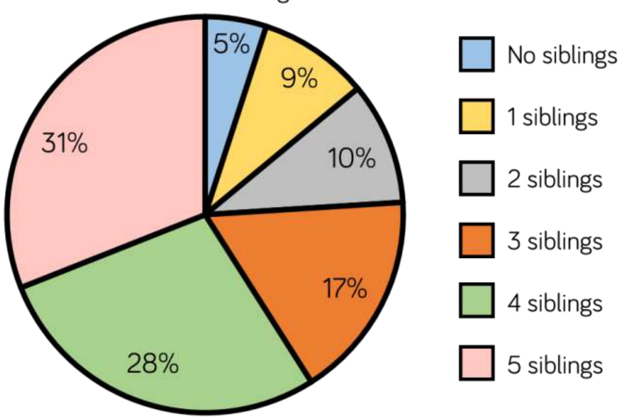


Pie Charts With Percentages

Reasoning and Problem Solving

15 people in this survey have no siblings. Use this information to work out how many people took part in the survey altogether.

Number of siblings

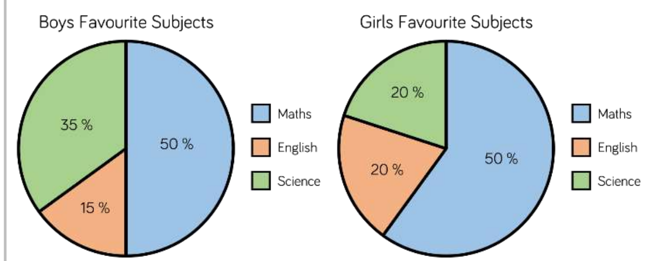


Now work out how many people each segment of the pie chart is worth.

Can you represent the information in a table?

No siblings	15
1 sibling	27
2 siblings	30
3 siblings	51
4 siblings	84
5 siblings	93
Total	300

120 boys and 100 girls were asked which was their favourite subject. Here are the results:



Jack says:

More girls prefer Maths than boys because 60 % is bigger than 50 %.

Do you agree? Explain why.

Jack is incorrect because the same amount of girls and boys like maths.

Boys:
50 % of 120 = 60

Girls:
60 % of 100 = 60

Draw Pie Charts

Notes and Guidance

Pupils will build on angles around a point totalling 360 degrees to know that this represents 100 % of the data within a pie chart.

From this, they will construct a pie chart, using a protractor to measure the angles. A "standard" protractor has radius 5 cm, so if circles of this radius are drawn, it is easier to construct the angles.

Mathematical Talk

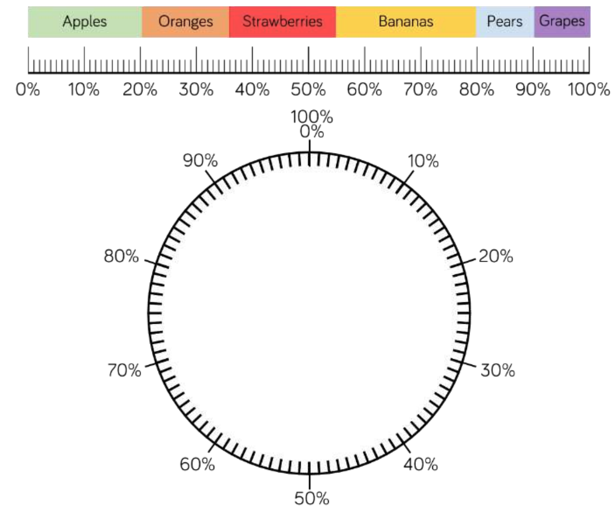
How many degrees are there around a point? How will this help us construct a pie chart?

If the total frequency is ____ , how will we work out the number of degrees representing each sector?

If 180° represents 15 pupils. How many people took part in the survey? Explain why.

Varied Fluency

Construct a pie chart using the data shown in this percentage bar model.



A survey was conducted to show how children in Class 6 travelled to school.

Draw a pie chart to represent the data.

Type of transport	Number of children	Convert to degrees
Car	12	12 × 10 = 120°
Bike	7	
Walk	8	
Bus	5	
Scooter	4	
Total	36	360°

Draw Pie Charts

Reasoning and Problem Solving

A survey was conducted to work out Year 6's favourite sport. Work out the missing information and then construct a pie chart.

Favourite sport	Number of children	Convert to degrees
Football	10	
Tennis	18	
Rugby		$\times 6 = 90°$
Swimming	6	$6 \times 6 = 36°$
Cricket		$\times 6 = 42°$
Golf	4	$4 \times 6 = 24°$
Total	60	360°



Children will then use this to draw a pie chart.

Favourite sport	Number of children	Convert to degrees
Football	10	$10 \times 6 = 60°$
Tennis	18	$18 \times 6 = 108°$
Rugby	15	$15 \times 6 = 90°$
Swimming	6	$6 \times 6 = 36°$
Cricket	7	$7 \times 6 = 42°$
Golf	4	$4 \times 6 = 24°$
Total	60	360°

A restaurant was working out which Sunday dinner was the most popular. Use the data to construct a pie chart.

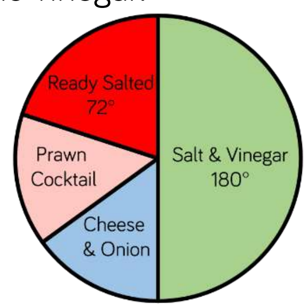
Dinner choice	Frequency	Convert to degrees
Chicken	11	
Pork	8	
Lamb	6	
Beef	9	
Vegetarian	6	
Total	40	

Children will then use this table to draw a pie chart.

Dinner choice	Frequency	Convert to degrees
Chicken	11	$11 \times 9 = 99°$
Pork	8	$8 \times 9 = 72°$
Lamb	6	$6 \times 9 = 54°$
Beef	9	$9 \times 9 = 81°$
Vegetarian	6	$6 \times 9 = 54°$
Total	40	360°

Miss Jones is carrying out a survey in class about favourite crisp flavours. 15 pupils chose salt and vinegar.

How many fewer people chose ready salted?



15 pupils = 180°
$180 \div 15 = 12$
12° = 1 pupil
$72 \div 12 = 6$ pupils
$15 - 6 = 9$
9 fewer students chose ready salted over salt and vinegar.

The Mean

Notes and Guidance

Children will apply their addition and division skills to calculate the mean average in a variety of contexts. They could find the mean by sharing equally or using the formula:
Mean = Total ÷ number of items.
Once children understand how to calculate the mean of a simple set of data, allow children time to investigate missing data when given the mean.

Mathematical Talk

What would the total be? If we know the total, how can we calculate the mean?

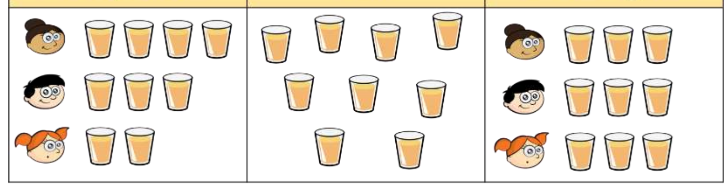
Do you think calculating the mean age of the family is a good indicator of their actual age? Why? *(Explore why this isn't helpful).*

When will the mean be useful in real life?

Varied Fluency

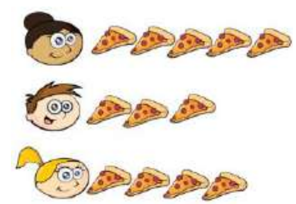
Here is a method to find the mean.

No. of glasses of juice drunk by 3 friends	Total glasses of juice drank	If each friend drank the same no. of glasses



The mean number of glasses of juice drunk is 3

Use this method to calculate the mean average for the number of slices of pizza eaten by each child.



Calculate the mean number of crayons:

Crayon colour	Amount
Blue	14
Green	11
Red	10
Yellow	9

Hassan is the top batsman for the cricket team. His scores over the year are: 134, 60, 17, 63, 38, 84, 11
Calculate the mean number of runs Hassan scored.

The Mean

Reasoning and Problem Solving

The mean number of goals scored in 6 football matches was 4.
Use this information to calculate how many goals were scored in the 6th match:

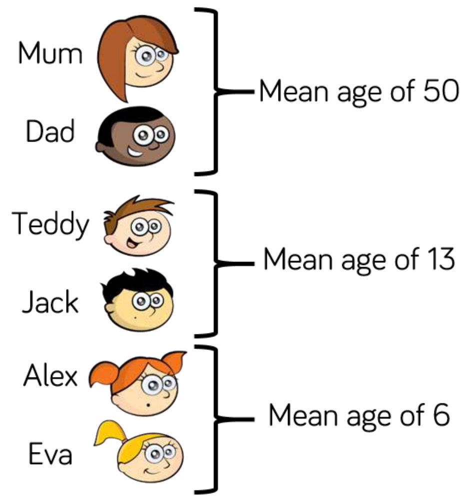
Match number	Number of goals
1	8
2	4
3	6
4	2
5	1
6	

As the mean is 4, the total must be
$6 \times 4 = 24$.
The missing number of goals is 3

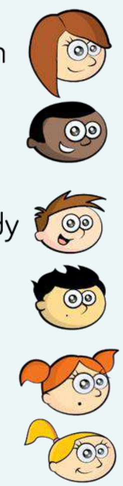
Three football teams each play 10 matches over a season. The mean number of goals scored by each team was 2.
How many goals might the teams have scored in each match?
How many solutions can you find?

Any sets of 10 numbers that total 20 e.g.
2, 2, 2, 2, 2, 2, 2, 2, 2 and 2
3, 1, 4, 5, 3, 1, 3, 0, 0 and 0 etc.

Work out the age of each member of the family if:
Mum is 48 years old.
Teddy is 4 years older than Jack and 7 years older than Alex.



Calculate the mean age of the whole family.



Mum 48
Dad 52
Teddy 15
Jack 11
Alex 8
Eva 4

23

Summer Scheme of Learning

Year 6

#MathsEveryoneCan

2020-21

White Rose Maths

New for 2020/21

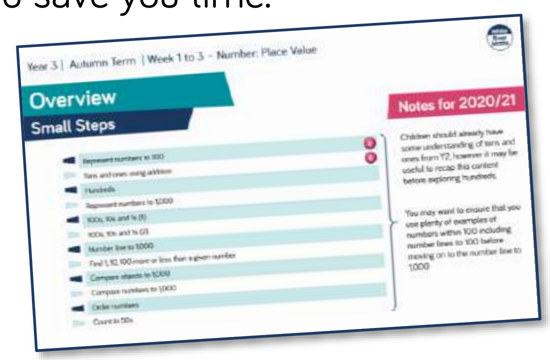
2020 will go down in history. The world has changed for all of us.

We want to do as much as we can to support children, teachers, parents and carers in these very uncertain times.

We have amended our schemes for 2020/21 to:

- ★ highlight key teaching points
- ★ recap essential content that children may have forgotten
- ★ flag any content that you might not have covered during the school closures period.

We hope these changes will add further value to the schemes and save you time.



Lesson-by-lesson overviews

We've always been reluctant to produce lesson-by-lesson overviews as every class is individual and has different needs. However, many of you have said that if blended learning becomes a key feature of school life next year, a weekly plan with linked content and videos could be really useful.

As always, we've listened! We've now produced a complete lesson-by-lesson overview for Y1 to Y9 that schools can use or adapt as they choose. Each lesson will be linked to a free-to-use home learning video, and for premium subscribers, a worksheet. This means that you can easily assign work to your class, whether they are working at home or in school.

Inevitably, this lesson-by-lesson structure won't suit everyone, but if it works for you, then please do make use of this resource as much as you wish.

Teaching for Mastery

These overviews are designed to support a mastery approach to teaching and learning and have been designed to support the aims and objectives of the new National Curriculum.

The overviews:

- have number at their heart. A large proportion of time is spent reinforcing number to build competency
- ensure teachers stay in the required key stage and support the ideal of depth before breadth.
- ensure students have the opportunity to stay together as they work through the schemes as a whole group
- provide plenty of opportunities to build reasoning and problem solving elements into the curriculum.

For more guidance on teaching for mastery, visit the NCETM website:

https://www.ncetm.org.uk/resources/47230

Concrete - Pictorial - Abstract

We believe that all children, when introduced to a new concept, should have the opportunity to build competency by taking this approach.

Concrete – children should have the opportunity to use concrete objects and manipulatives to help them understand what they are doing.

Pictorial – alongside this children should use pictorial representations. These representations can then be used to help reason and solve problems.

Abstract – both concrete and pictorial representations should support children's understanding of abstract methods.

Need some CPD to develop this approach? Visit www.whiterosemaths.com for find a course right for you.

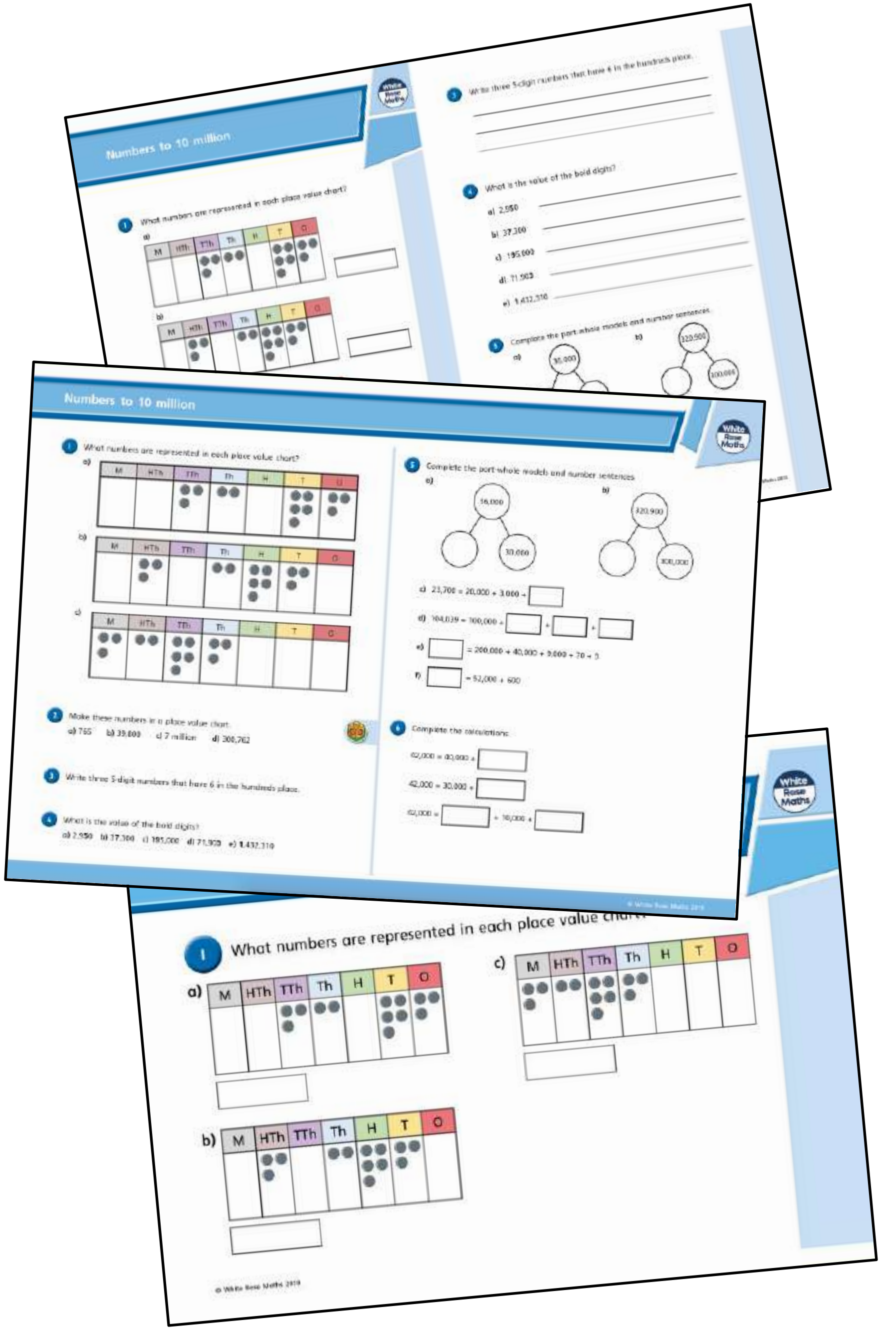
Supporting resources

We have produced supporting resources for every small step from Year 1 to Year 11.

The worksheets are provided in three different formats:

- Write on worksheet – ideal for children to use the ready made models, images and stem sentences.
- Display version – great for schools who want to cut down on photocopying.
- PowerPoint version – one question per slide. Perfect for whole class teaching or mixing questions to make your own bespoke lesson.

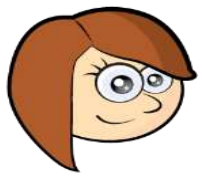
For more information visit our online training and resources centre resources.whiterosemaths.com or email us directly at support@whiterosemaths.com



Meet the Characters

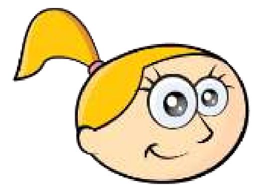
Children love to learn with characters and our team within the scheme will be sure to get them talking and reasoning about mathematical concepts and ideas. Who's your favourite?

Teddy

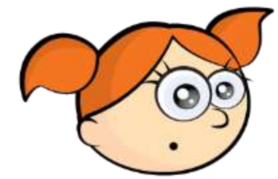


Rosie

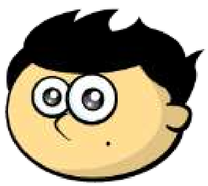
Mo



Eva

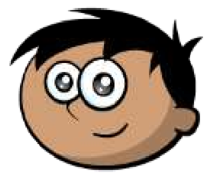


Alex

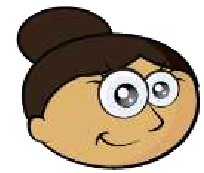


Jack

Whitney



Amir



Dora

Tommy

Dexter

Ron

Annie

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	Number: Place Value		Number: Addition, Subtraction, Multiplication and Division					Number: Fractions				Geometry: Position and Direction
Spring	Number: Decimals		Number: Percentages		Number: Algebra		Measurement: Converting Units	Measurement: Perimeter, Area and Volume		Number: Ratio		Consolidation
Summer	Statistics		Geometry: Properties of shape			Consolidation and themed projects						

Summer - Block 1

Statistics

Overview

Small Steps

- Read and interpret line graphs
- Draw line graphs
- Use line graphs to solve problems
- Circles
- Read and interpret pie charts
- Pie charts with percentages
- Draw pie charts
- The mean

Notes for 2020/21

Originally this had been planned in for the end of the Spring term. Due to SATs being cancelled and therefore time gained for year 6 teachers, this can now be covered in more detail at the start of the summer term.

There will be more opportunity to draw pie charts in the next block when children recap measuring and drawing angles.

Read and Interpret Line Graphs

Notes and Guidance

Children will build on their experience of interpreting data in context from Year 5, using their knowledge of scales to read information accurately. Examples of graphs are given but it would be useful if real data from across the curriculum e.g. Science, was also used. Please note that line graphs represent continuous data not discrete data. Children need to read information accurately, including where more than one set of data is on the same graph.

Mathematical Talk

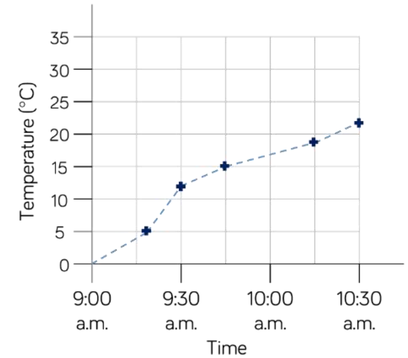
Where might you see a line graph used in real life?

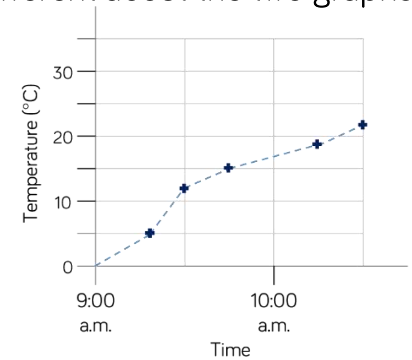
Why is the 'Water Consumption' graph more difficult to interpret?

How can you make sure that you read the information accurately?

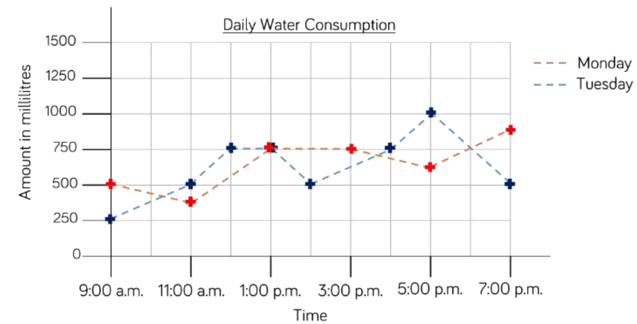
Varied Fluency

What is the same and what is different about the two graphs?





Here is a graph showing daily water consumption over two days.



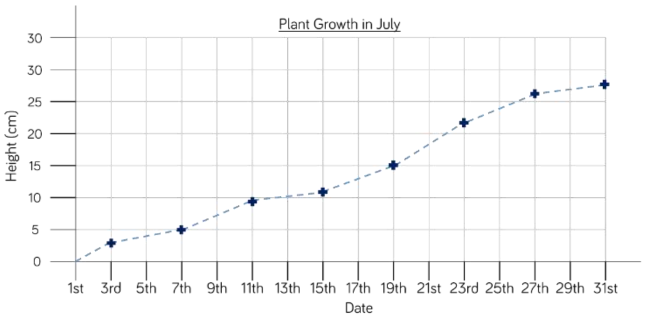
At what times of the day was the same amount of water consumed on Monday and Tuesday?

Was more water consumed at 2 p.m. on Monday or Tuesday morning? How much more?

Read and Interpret Line Graphs

Reasoning and Problem Solving

Eva has created a graph to track the growth of a plant in her house.



Eva recorded the following facts about the graph.

a) On the 9th of July the plant was about 9 cm tall.

b) Between the 11th and 19th July the plant grew about 5 cm.

c) At the end of the month the plant was twice as tall as it had been on the 13th .

Can you spot and correct Eva's mistakes?

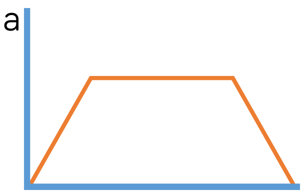
a) On the 9th July a more accurate measurement would be 7.5 cm.

b) Correct.

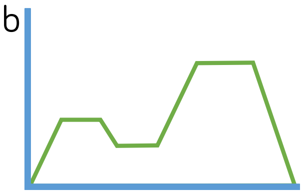
c) On the 31st the plant was approximately 28 cm tall, but on the 13th it was only 10 cm which is not half of 28 cm. The plant was closer to 14 cm on the 17th July.

Write a story and 3 questions for each of the 3 graphs below.

a



b



c

Possible context for each story:

a) A car speeding up, travelling at a constant speed, then slowing down.

b) The height above sea level a person is at during a walk.

c) Temperature in an oven when you are cooking something.

Draw Line Graphs

Notes and Guidance

Children will build on their experience of reading and interpreting data in order to draw their own line graphs.

Although example contexts are given, it would be useful if children can see real data from across the curriculum.

Children will need to decide on the most appropriate scales and intervals to use depending on the data they are representing.

Mathematical Talk

What will the x-axis represent? What intervals will you use?

What will the y-axis represent? What intervals will you use?

How will you make it clear which line represents which set of data?

Why is it useful to have both sets of data on one graph?

Varied Fluency

- This table shows the height a rocket reached between 0 and 60 seconds.

  Create a line graph to represent the information.

Time (seconds)	Height (metres)
0	0
10	8
20	15
30	25
40	37
50	50
60	70

- The table below shows the population in the UK and Australia from 1990 to 2015.

	1990	1995	2000
UK	57,200,000	58,000,000	58,900,000
Australia	17,000,000	18,000,000	19,000,000
	2005	2010	2015
UK	60,300,000	63,300,000	65,400,000
Australia	20,200,000	22,100,000	23,800,000

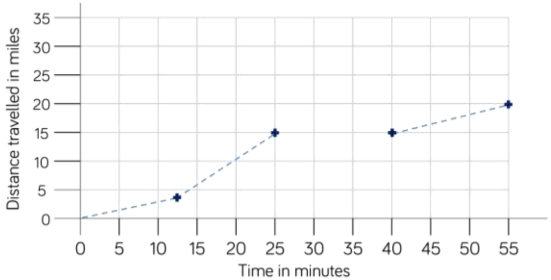
  Create one line graph to represent the population in both countries. Create three questions to ask your friend about your completed graph.

White Rose Maths

Draw Line Graphs

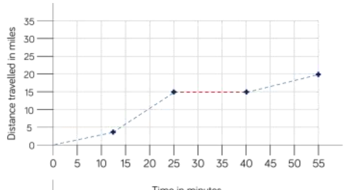
Reasoning and Problem Solving

This graph shows the distance a car travelled.

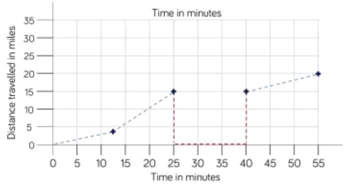


Rosie and Jack were asked to complete the graph to show the car had stopped.
Here are their completed graphs.

Rosie:



Jack:



Who has completed the graph correctly? Explain how you know.

Rosie has completed the graph correctly. The car has still travelled 15 miles in total, then stopped for 15 minutes before carrying on.

This table shows the distance a lorry travelled during the day.

Time	Distance in miles
7.00 a.m.	10
8.00 a.m.	28
9.00 a.m.	42
10.00 a.m.	58
11.00 a.m.	70
12.00 a.m.	95
1.00 p.m.	95
2.00 p.m.	118

Create a line graph to represent the information, where the divisions along the x-axis are every two hours.
Create a second line graph where the divisions along the x-axis are every hour.
Compare your graphs. Which graph is more accurate?
Would a graph with divisions at each half hour be even more accurate?

Children may find that the second line graph is easier to draw and interpret as it matches the data given directly.

They may discuss that it would be difficult to draw a line graph showing half hour intervals, as we cannot be sure the distance travelled at each half hour.

Line Graphs Problems

Notes and Guidance

Once children can read, interpret and draw lines graphs they need to be able to use line graphs to solve problems.

Children need to use their knowledge of scales to read information accurately. They need to be exposed to graphs that show more than one set of data.

At this point, children should be secure with the terms x and y axis, frequency and data.

Mathematical Talk

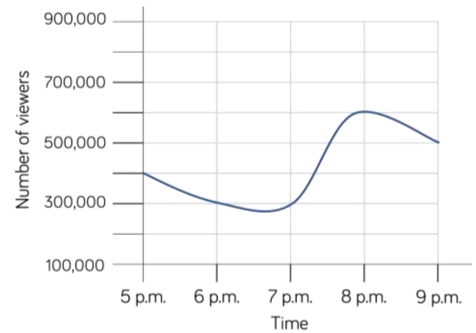
What do you notice about the scale on the vertical axis? Why might it be misleading?
What other scale could you use?

How is the information organised? Is it clear?
What else does this graph tell you? What does it not tell you?

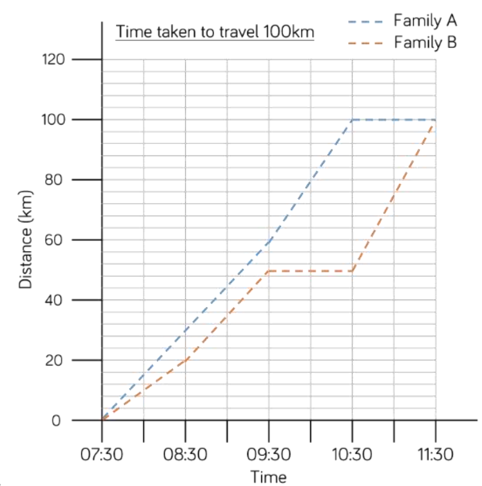
How can you calculate ________?
Why would this information be placed on a line graph and not a different type of graph?

Varied Fluency

- Ron and Annie watched the same channel, but at different times.
The graph shows the number of viewers at different times.
Ron watched 'Chums' at 5 p.m. Annie watched 'Countup' at 8 p.m.



What was the difference between the number of viewers at the start of each programme? What was the difference in the number of viewers between 6 p.m. and 8 p.m.? Which time had twice as many viewers as 6 p.m.?

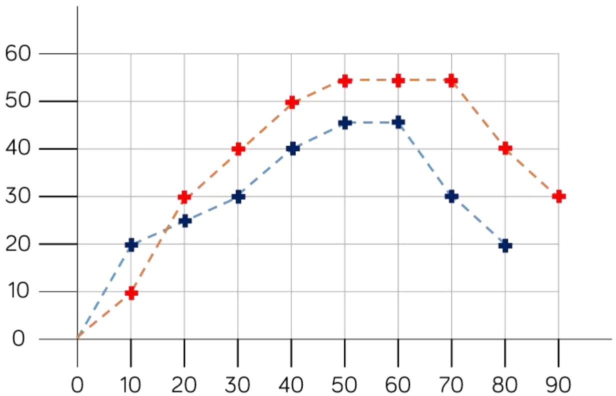
- 

Two families were travelling to Bridlington for their holidays. They set off at the same time but arrived at different times.
What time did family A arrive?
How many km had each family travelled at 08:45?
Which family stopped midway through their journey?
How much further had they left to travel?

Line Graphs Problems

Reasoning and Problem Solving

What could this graph be showing?



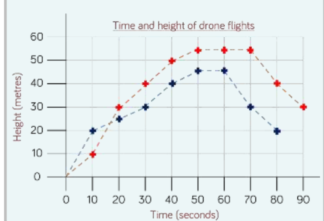
Label the horizontal and vertical axes to show this.

Is there more than one way to label the axes?

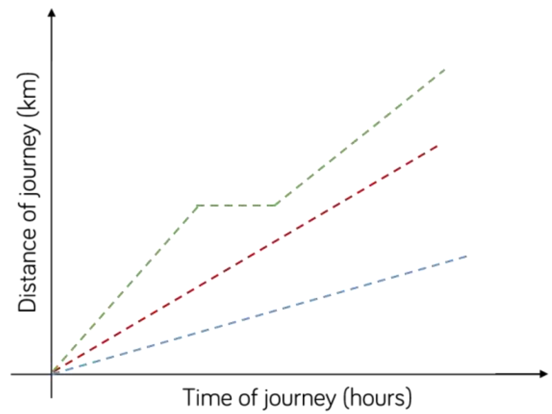
Possible response:

This graph shows the height of two drones and the time they were in the air.

For example:



The graph below shows some of Mr Woolley's journeys.



What is the same and what is different about each of these journeys?

What might have happened during the green journey?

Possible responses:

All the journeys were nearly the same length of time.

The journeys were all different distances.

The red and blue journey were travelling at constant speeds but red was travelling quicker than blue.

During the green journey, Mr Woolley might have been stuck in traffic or have stopped for a rest.

Circles

Notes and Guidance

Children will illustrate and name parts of circles, using the words radius, diameter, centre and circumference confidently.

They will also explore the relationship between the radius and the diameter and recognise the diameter is twice the length of the radius.

Mathematical Talk

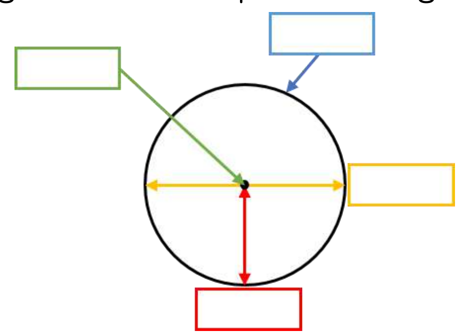
Why is the centre important?

What is the relationship between the diameter and the radius? If you know one of these, how can you calculate the other?

Can you use the vocabulary of a circle to describe and compare objects in the classroom?

Varied Fluency

Using the labels complete the diagram:



Radius

Diameter

Centre

Circumference

Find the radius or the diameter for each object below:

The radius is ___. The diameter is ___. I know this because ___.

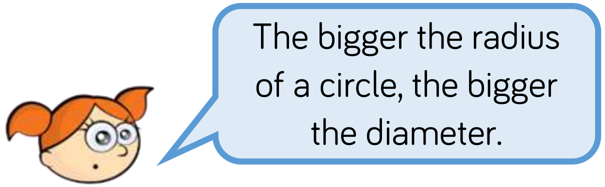
Complete the table:

Radius	Diameter
26 cm	
	37 mm
2.55 m	
	99 cm
	19.36 cm

Circles

Reasoning and Problem Solving

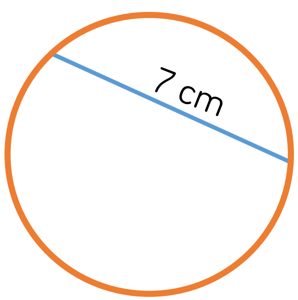
Alex says:



Do you agree? Explain your reasoning.

I agree with Alex because the diameter is always twice the length of the radius.

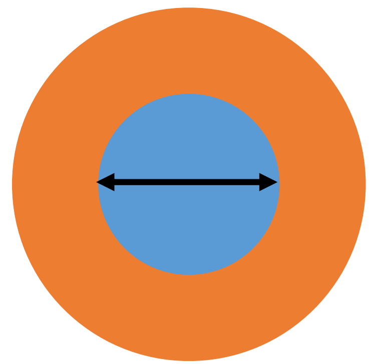
Spot the mistake!
Tommy has measured and labelled the diameter of the circle below.
He thinks that the radius of this circle will be 3.5 cm.



Is Tommy right? Explain why.

Tommy has measured the diameter inaccurately because the diameter always goes through the centre of the circle from one point on the circumference to another.

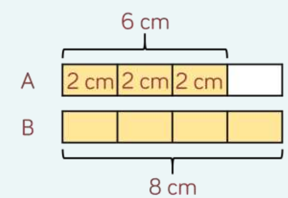
Here are 2 circles. Circle A is blue; Circle B is orange. The diameter of Circle A is $\frac{3}{4}$ the diameter of Circle B.



If the diameter of Circle B is 12 cm, what is the diameter of Circle A?
If the diameter of Circle A is 12 cm, what is the radius of Circle B?
If the diameter of Circle B is 6 cm, what is the diameter of Circle A?
If the diameter of Circle A is 6 cm, what is the radius of Circle B?

a) 9 cm
b) 16 cm
c) 4.5 cm
d) 8 cm

A bar model may support children in working these out e.g.



Read and Interpret Pie Charts

Notes and Guidance

Children will build on their understanding of circles to start interpreting pie charts. They will understand how to calculate fractions of amounts to interpret simple pie charts.

Children should understand what the whole of the pie chart represents and use this when solving problems.

Mathematical Talk

What does the whole pie chart represent? What does each colour represent?

Do you recognise any of the fractions? How can you use this to help you?

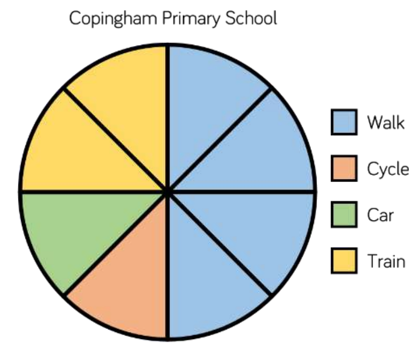
What's the same and what's different about the favourite drinks pie charts?

What other questions could you ask about the pie chart?

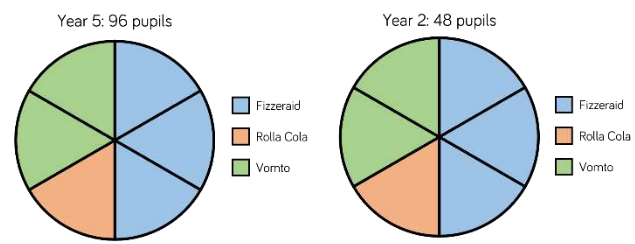
Varied Fluency

- There are 600 pupils at Copingham Primary school. Work out how many pupils travel to school by:

  a) Train
  b) Car
  c) Cycling
  d) Walking



- Classes in Year 2 and Year 5 were asked what their favourite drink was. Here are the results:



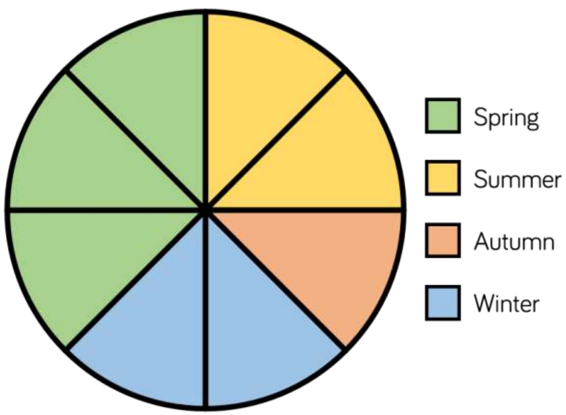
  What fraction of pupils in Year 5 chose Fizzeraid?
  How many children in Year 2 chose Rolla Cola?
  How many more children chose Vomto than Rolla Cola in Year 2?
  What other questions could you ask?

Read and Interpret Pie Charts

Reasoning and Problem Solving

In a survey people were asked what their favourite season of the year was. The results are shown in the pie chart below. If 48 people voted summer, how many people took part in the survey?

Our favourite time of year

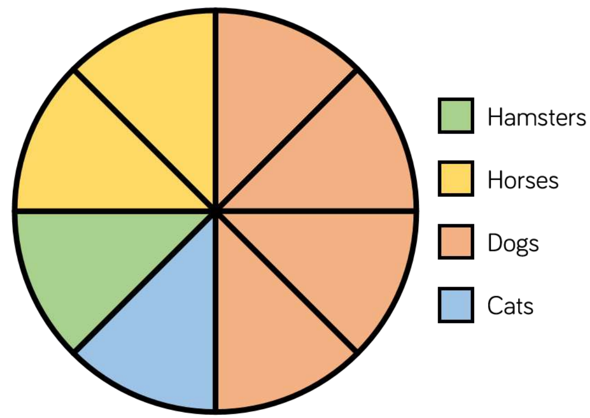


Explain your method.

Summer is a quarter of the whole pie chart and there are 4 quarters in a whole, so $48 \times 4 = 184$ people in total.

96 people took part in this survey.

Our favourite pets



How many people voted for cats?

$\frac{3}{8}$ of the people who voted for dogs were male. How many females voted for dogs?

What other information can you gather from the pie chart?

Write some questions about the pie chart for your partner to solve.

$\frac{1}{2}$ of $96 = 48$

$\frac{1}{4}$ of $96 = 24$

$\frac{1}{8}$ of $96 = 12$

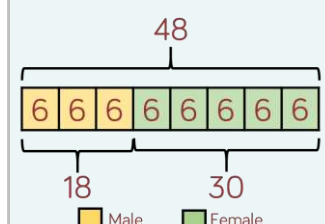
12 people voted cats.

48 people voted dogs.

$\frac{1}{8}$ of $48 = 6$

$6 \times 3 = 18.$

18 females voted for dogs.



Pie Charts With Percentages

Notes and Guidance

Children will apply their understanding of calculating percentages of amounts to interpret pie charts.

Children know that the whole of the pie chart totals 100 %.

Encourage children to recognise fractions in order to read the pie chart more efficiently.

Mathematical Talk

How did you calculate the percentage? What fraction knowledge did you use?

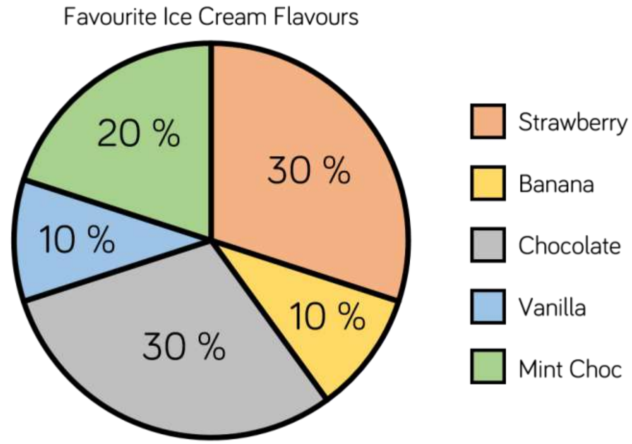
How else could you find the difference between Chocolate and Mint Chocolate?

If you know 5 % of a number, how can you work out the whole number?

If you know what 5 % is, what else do you know?

Varied Fluency

150 children voted for their favourite ice cream flavours. Here are their results:

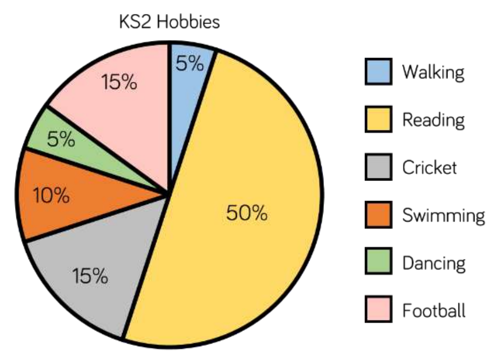


How many people voted for Vanilla?

How many more people voted for Chocolate than Mint Chocolate Chip?

How many people chose Chocolate, Banana and Vanilla altogether?

There are 200 pupils in Key Stage 2 who chose their favourite hobbies.

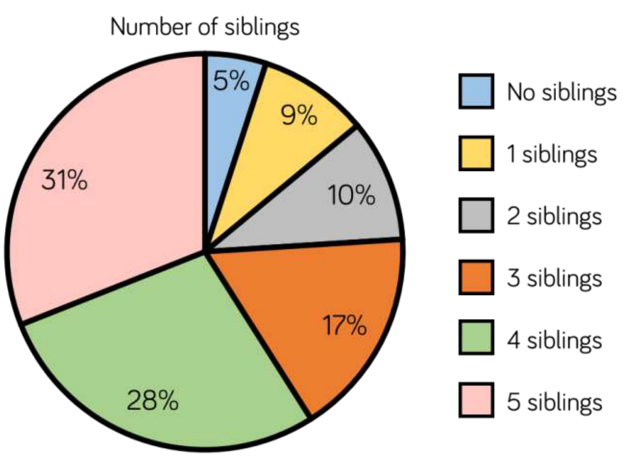


How many pupils chose each hobby?

Pie Charts With Percentages

Reasoning and Problem Solving

15 people in this survey have no siblings. Use this information to work out how many people took part in the survey altogether.

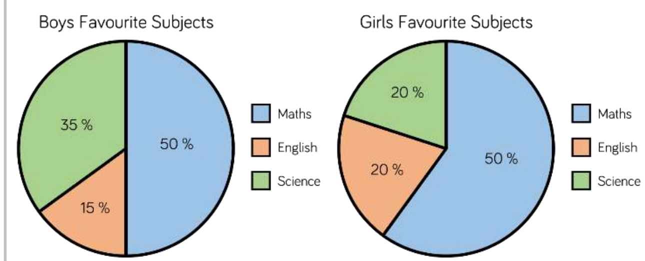


Now work out how many people each segment of the pie chart is worth.

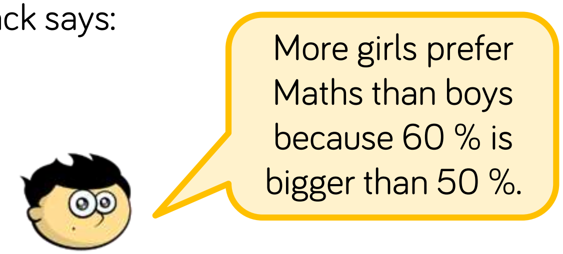
Can you represent the information in a table?

No siblings	15
1 sibling	27
2 siblings	30
3 siblings	51
4 siblings	84
5 siblings	93
Total	300

120 boys and 100 girls were asked which was their favourite subject. Here are the results:



Jack says:



Do you agree? Explain why.

Jack is incorrect because the same amount of girls and boys like maths.

Boys:
50 % of 120 = 60

Girls:
60 % of 100 = 60

Draw Pie Charts

Notes and Guidance

Pupils will build on angles around a point totalling 360 degrees to know that this represents 100 % of the data within a pie chart.

From this, they will construct a pie chart, using a protractor to measure the angles. A “standard” protractor has radius 5 cm, so if circles of this radius are drawn, it is easier to construct the angles.

Mathematical Talk

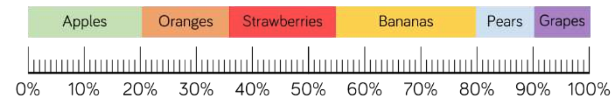
How many degrees are there around a point? How will this help us construct a pie chart?

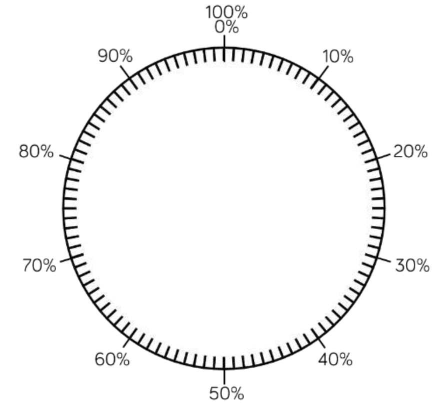
If the total frequency is ____ , how will we work out the number of degrees representing each sector?

If 180° represents 15 pupils. How many people took part in the survey? Explain why.

Varied Fluency

Construct a pie chart using the data shown in this percentage bar model.





A survey was conducted to show how children in Class 6 travelled to school.

Draw a pie chart to represent the data.

Type of transport	Number of children	Convert to degrees
Car	12	$12 \times 10 = 120°$
Bike	7	
Walk	8	
Bus	5	
Scooter	4	
Total	36	360°

Draw Pie Charts

Reasoning and Problem Solving

A survey was conducted to work out Year 6's favourite sport. Work out the missing information and then construct a pie chart.

Favourite sport	Number of children	Convert to degrees
Football	10	
Tennis	18	
Rugby		× 6 = 90°
Swimming	6	6 × 6 = 36°
Cricket		× 6 = 42°
Golf	4	4 × 6 = 24°
Total	60	360°



Children will then use this to draw a pie chart.

Favourite sport	Number of children	Convert to degrees
Football	10	10 × 6 = 60°
Tennis	18	18 × 6 = 108°
Rugby	15	15 × 6 = 90°
Swimming	6	6 × 6 = 36°
Cricket	7	7 × 6 = 42°
Golf	4	4 × 6 = 24°
Total	60	360°

A restaurant was working out which Sunday dinner was the most popular. Use the data to construct a pie chart.

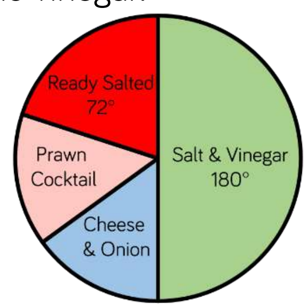
Dinner choice	Frequency	Convert to degrees
Chicken	11	
Pork	8	
Lamb	6	
Beef	9	
Vegetarian	6	
Total	40	

Children will then use this table to draw a pie chart.

Dinner choice	Frequency	Convert to degrees
Chicken	11	11 × 9 = 99°
Pork	8	8 × 9 = 72°
Lamb	6	6 × 9 = 54°
Beef	9	9 × 9 = 81°
Vegetarian	6	6 × 9 = 54°
Total	40	360°

Miss Jones is carrying out a survey in class about favourite crisp flavours. 15 pupils chose salt and vinegar.

How many fewer people chose ready salted?



15 pupils = 180°
180 ÷ 15 = 12
12° = 1 pupil
72 ÷ 12 = 6 pupils
15 − 6 = 9
9 fewer students chose ready salted over salt and vinegar.

The Mean

Notes and Guidance

Children will apply their addition and division skills to calculate the mean average in a variety of contexts. They could find the mean by sharing equally or using the formula:
Mean = Total ÷ number of items.
Once children understand how to calculate the mean of a simple set of data, allow children time to investigate missing data when given the mean.

Mathematical Talk

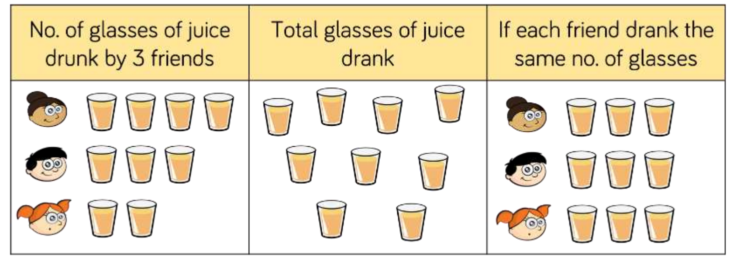
What would the total be? If we know the total, how can we calculate the mean?

Do you think calculating the mean age of the family is a good indicator of their actual age? Why? *(Explore why this isn't helpful).*

When will the mean be useful in real life?

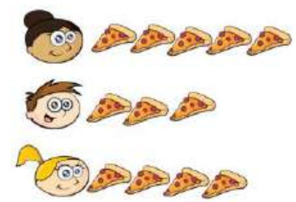
Varied Fluency

Here is a method to find the mean.



The mean number of glasses of juice drunk is 3

Use this method to calculate the mean average for the number of slices of pizza eaten by each child.



Calculate the mean number of crayons:

Crayon colour	Amount
Blue	14
Green	11
Red	10
Yellow	9

Hassan is the top batsman for the cricket team. His scores over the year are: 134, 60, 17, 63, 38, 84, 11
Calculate the mean number of runs Hassan scored.

The Mean

Reasoning and Problem Solving

The mean number of goals scored in 6 football matches was 4.
Use this information to calculate how many goals were scored in the 6th match:

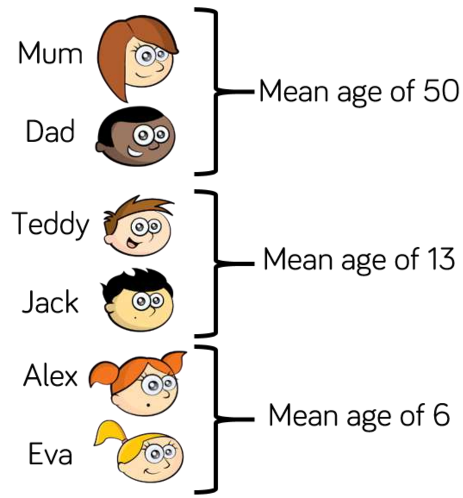
Match number	Number of goals
1	8
2	4
3	6
4	2
5	1
6	

As the mean is 4, the total must be
$6 \times 4 = 24$.
The missing number of goals is 3

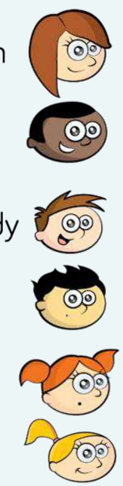
Three football teams each play 10 matches over a season. The mean number of goals scored by each team was 2.
How many goals might the teams have scored in each match?
How many solutions can you find?

Any sets of 10 numbers that total 20 e.g.
2, 2, 2, 2, 2, 2, 2, 2, 2 and 2
3, 1, 4, 5, 3, 1, 3, 0, 0 and 0 etc.

Work out the age of each member of the family if:
Mum is 48 years old.
Teddy is 4 years older than Jack and 7 years older than Alex.



Calculate the mean age of the whole family.



Mum	48
Dad	52
Teddy	15
Jack	11
Alex	8
Eva	4

23

Summer - Block 2

Properties of Shape

Overview

Small Steps

- Measure with a protractor
- Draw lines and angles accurately (R)
- Introduce angles
- Angles on a straight line (R)
- Angles around a point (R)
- Calculate angles
- Vertically opposite angles
- Angles in a triangle
- Angles in a triangle – special cases
- Angles in a triangle – missing angles
- Angles in special quadrilaterals
- Angles in regular polygons
- Draw shapes accurately
- Draw nets of 3-D shapes

Notes for 2020/21

In this block children will build on learning from year 5 to look at properties of shape in detail, specifically angles.

There is time available after this block so it can span a longer period of time if needed.

Consider recapping the drawing of pie charts from the previous block when working with protractors.

Measure with a Protractor

Notes and Guidance

This step revisits measuring angles using a protractor from Year 5
Children recap how to line up the protractor accurately, and identify which side of the scale to read. They link this to their understanding of angle sizes.
Children read the measurement and practise measuring angles given in different orientations.
Angles are also related to compass points.

Mathematical Talk

Can we name and describe the 4 different types of angles? (right angle, obtuse, acute, reflex)

What unit do we use to measure angles?

Does it matter which side of the protractor I use?

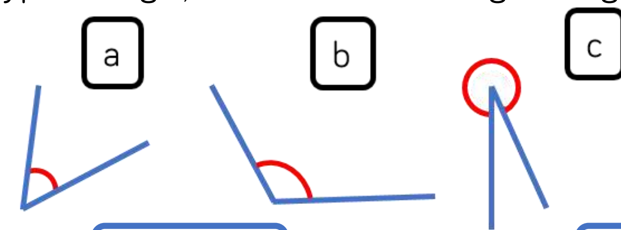
What mistakes could we make when measuring with a protractor?

How would I measure a reflex angle?

Look at a compass, what angles can we identify using the compass?

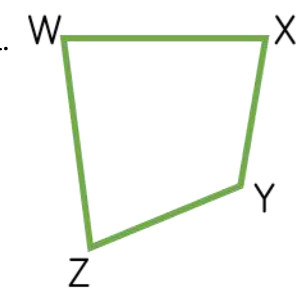
Varied Fluency

- Identify the type of angle, and measure the angle using a protractor.



Angle ☐ is an ☐ angle. It measures ☐

- Estimate, then measure each of the angles at the vertices of the quadrilateral.

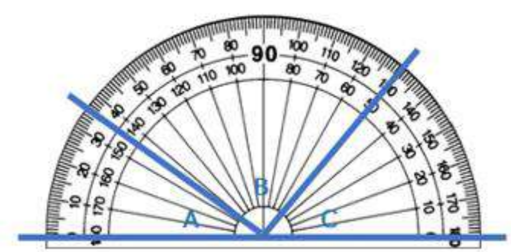


W: ☐ X: ☐

Y: ☐ Z: ☐

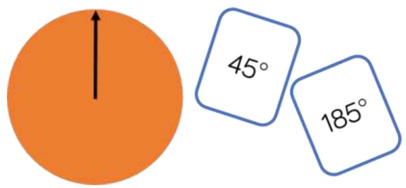
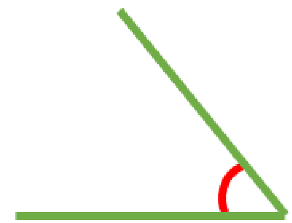
- Work out the size of each angle.

Explain how you found your answers.



Measure with a Protractor

Reasoning and Problem Solving

Cut out a circle and draw a line from the centre to the edge. Add a spinner in the centre.  Put the arrow in the starting position as shown above. Turn over a flash card with an angle on. Estimate the given angle by moving the spinner. Check how close you are using a protractor.	Children could work in pairs and get a partner to check the accuracy of the angles made.
Alex measures this angle:   She says it is 130° Explain what she has done wrong.	Alex is wrong because 130° is an obtuse angle and the angle indicated is acute. She has used the wrong scale on the protractor. She should have measured the angle to be 50°

Drawing Accurately

Notes and Guidance

Children need to draw lines correct to the nearest millimetre. They use a protractor to draw angles of a given size, and will need to be shown this new skill.

Children continue to develop their estimation skills whilst drawing and measuring lines and angles. They also continue to use precise language to describe the types of angles they are drawing.

Mathematical Talk

How many millimetres are in a centimetre?

How do we draw a line that measures ___?

Explain how to draw an angle.

What's the same and what's different about drawing angles of 80° and 100°?

How can I make this angle measure ___ but one of the lines have a length of ___?

Varied Fluency

Draw lines that measure:

4 cm and 5 mm	45 mm	4.5 cm

What's the same? What's different?

Draw:

- angles of 45° and 135°
- angles of 80° and 100°
- angles of 20° and 160°

What do you notice about your pairs of angles?

Draw:

- an acute angle that measures 60° with the arms of the angle 6 cm long
- an obtuse angle that measures 130° but less than 140° with the arms of the angle 6.5 cm long

Compare your angles with your partner's.

Drawing Accurately

Reasoning and Problem Solving

R

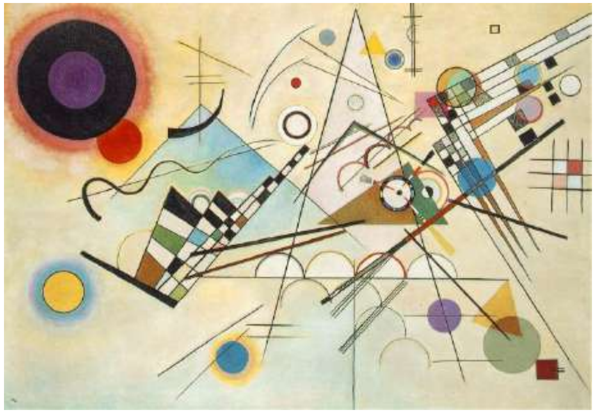
Draw a range of angles for a friend.
Estimate the sizes of the angles to order them from smallest to largest.
Measure the angles to see how close you were.

Always, sometimes or never true?

- Two acute angles next to each other make an obtuse angle.
- Half an obtuse angle is an acute angle.
- 180° is an obtuse angle

- Sometimes
- Always
- Never

Use Kandinsky's artwork to practice measuring lines and angles.



Create clues for your partner to work out which line or angle you have measured.

For example, "My line is horizontal and has an obtuse angle of 110° on it."

Introduce Angles

Notes and Guidance

Children build on their understanding of degrees in a right angle and make the connection that there are two right angles on a straight line and four right angles around a point.

Children should make links to whole, quarter, half and three-quarter turns and apply this in different contexts such as time and on a compass.

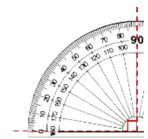
Mathematical Talk

If there are 90 degrees in one right angle, how many are there in two? What about three?

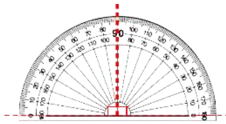
How many degrees are there in a quarter/half turn?

Between which two compass points can you see a right angle/half turn/three quarter turn?

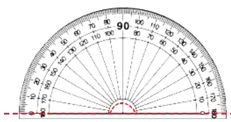
Varied Fluency



There are ☐ degrees in a right angle.



There are ☐ right angles on a straight line.



There are ☐ degrees on a straight line.

Complete the table.

Angle	Fraction of a full turn	Degrees
Right angle	$\frac{1}{4}$	90°
Straight line		
Three right angles		
Full turn		

Use a compass to identify how many degrees there are between:

- North & South (turning clockwise)
- South & East (turning anti-clockwise)
- North-East and South-West (turning clockwise)

Introduce Angles

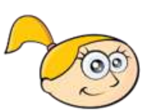
Reasoning and Problem Solving

Dora and Eva are asked how many degrees there are between North-West and South-West.

Dora says,

There are 90 degrees between NW and SW.

Eva says,



There are 270° between NW and SW.

Who do you agree with?
Explain why.

They are both correct. Dora measured anti-clockwise whereas Eva measured clockwise.

If it takes 60 minutes for the minute hand to travel all the way around the clock, how many degrees does the minute hand travel in:

- 7 minutes
- 12 minutes

How many minutes have passed if the minute hand has moved 162°?

$360 \div 60 = 6$ so the minute hand travels 6° per minute.
7 minutes : 42°
12 minutes : 72°

162° : 27 minutes

Always, sometimes, never.

W to S = 90 degrees
NE to SW = 180 degrees
E to SE in a clockwise direction > 90°

Sometimes
Always
Never

Angles on a Straight Line

Notes and Guidance

Children build on their knowledge of a right angle and recognise two right angles are equivalent to a straight line, or a straight line is a half of a turn.
Once children are aware that angles on a straight line add to 180 degrees, they use this to calculate missing angles on straight lines.
Part-whole and bar models may be used to represent missing angles.

Mathematical Talk

How many degrees are there in a right angle?

How many will there be in two right angles?

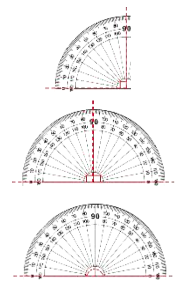
If we place two right angles together, what do we notice?

How can we calculate the missing angles?

How can we subtract a number from 180 mentally?

Varied Fluency

R

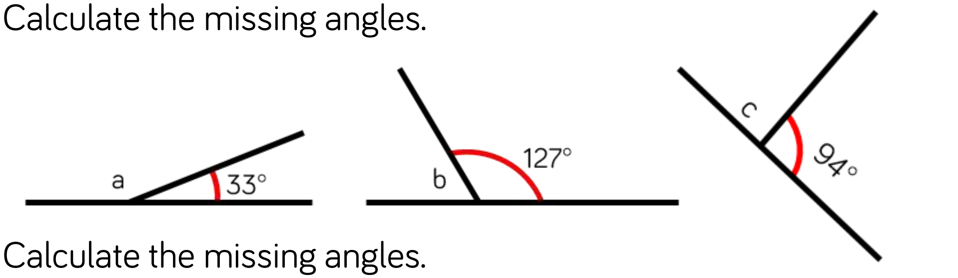


There are ______degrees in a right angle.

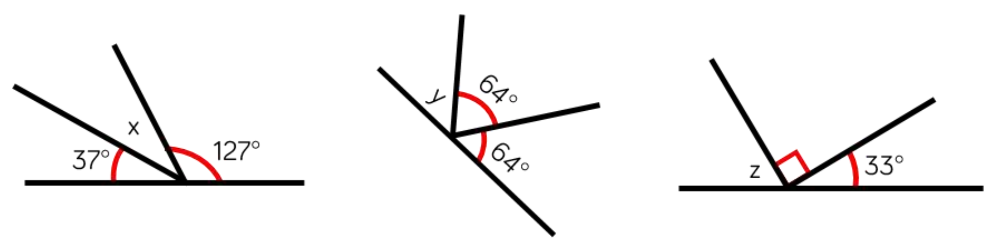
There are ______right angles on a straight line.

There are ______degrees on a straight line.

Calculate the missing angles.



Calculate the missing angles.



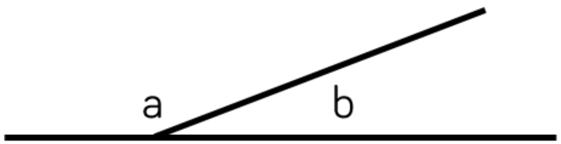
Is there more than one way to calculate the missing angles?

Angles on a Straight Line

Reasoning and Problem Solving

R

Here are two angles.



Angle b is a prime number between 40 and 50

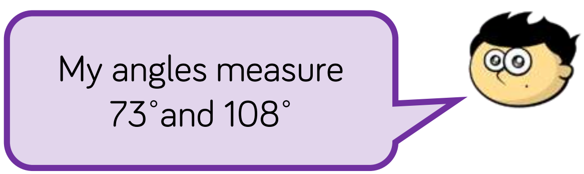
Use the clue to calculate what the missing angles could be.

b = 41°, a = 139°

b = 43°, a = 137°

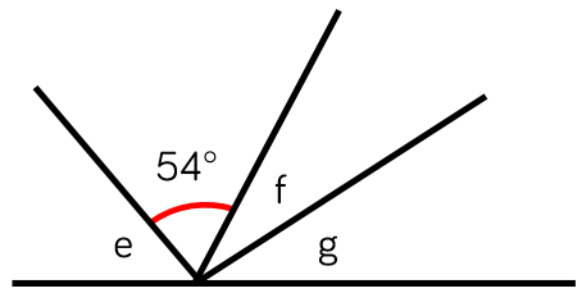
b = 47°, a = 133°

Jack is measuring two angles on a straight line.



Explain why at least one of Jack's angles must be wrong.

His angles total more than 180°.



- The total of angle f and g are the same as angle e
- Angle e is 9° more than the size of the given angle.
- Angle f is 11° more than angle g

Calculate the size of the angles.

Create your own straight line problem like this one for your partner.

e = 63°

f = 37°

g = 26 °

Angles around a Point

Notes and Guidance

Children need to know that there are 360 degrees in a full turn. This connects to their knowledge of right angles, full turns and compass points.

Children need to know when they should measure an angle and when they should calculate the size of angle from given facts.

Mathematical Talk

How many right angles are there in $\frac{1}{4}, \frac{1}{2}, \frac{3}{4}$ of a full turn?

If you know a half turn/full turn is 180/360 degrees, how can this help you calculate the missing angle?

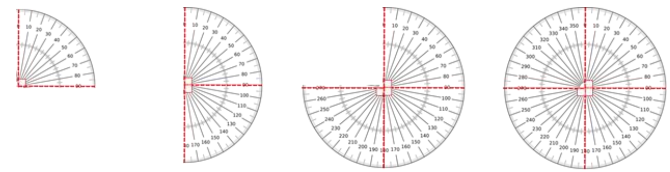
What is the most efficient way to calculate a missing angle? Would you use a mental or written method?

When you have several angles, is it better to add them first or to subtract them one by one?

Varied Fluency

R

- Complete the sentences.



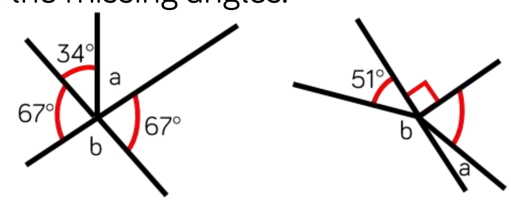
$\frac{1}{4}$ of a turn = 1 right angle = 90°

$\frac{1}{2}$ of a turn = __ right angles = _____°

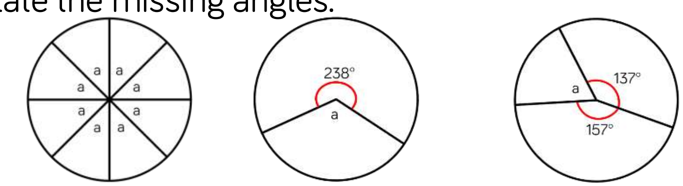
$\frac{\square}{\square}$ of a turn = 3 right angles = _____°

A full turn = __ right angles = _____°

- Calculate the missing angles.

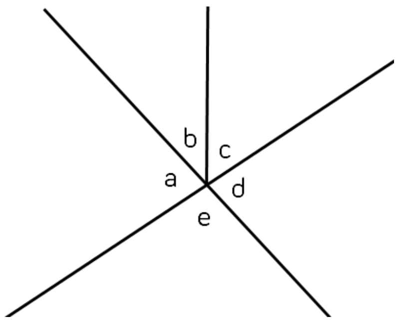


- Calculate the missing angles.



Angles around a Point

Reasoning and Problem Solving



$a + b + c + d + e = 360°$
$d + e = 180°$
Write other sentences about this picture.

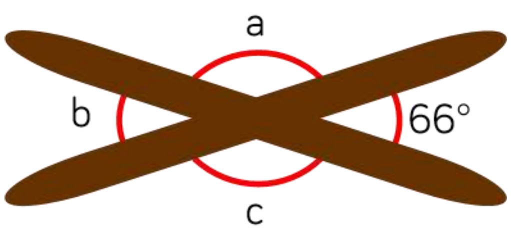
Various answers e.g.

$a + b + c = e + d$

$360° - e - d = 180°$

Etc.

Two sticks are on a table.
Without measuring, find the three missing angles.



$a = 114°$

$b = 66°$

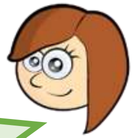
$c = 114°$

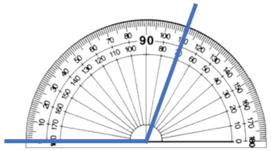
Eva says,

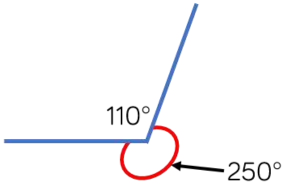
My protractor only goes to 180 degrees, so I can't draw reflex angles like 250 degrees.

Rosie says,

I know a full turn is 360 degrees so I can draw 110 degrees instead and have an angle of 250 degrees as well.







Use Rosie's method to draw angles of:
- 300°
- 200°
- 280°

Calculate Angles

Notes and Guidance

Children apply their understanding of angles in a right angle, angles on a straight line and angles around a point to calculate missing angles.

They should also recognise right angle notation and identify these on a diagram. Children then use this information to help them calculate unknown angles.

Mathematical Talk

What do we know about a and b? How do we know this?

Which angle fact might you need to use when answering this question?

Which angles are already given? How can we use this to calculate unknown angles?

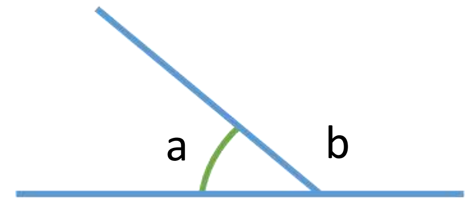
Varied Fluency

- a + b = □

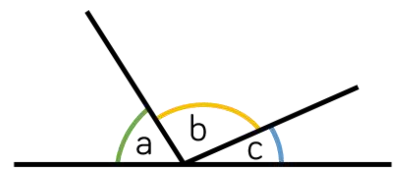
  b + a = □

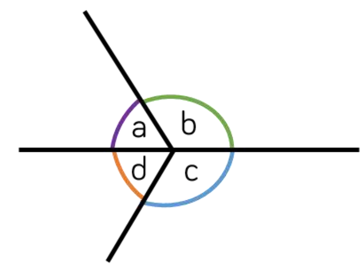
  □ − a = b

  □ − b = a

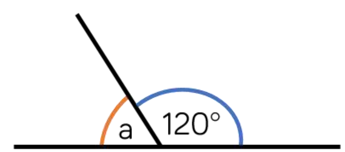


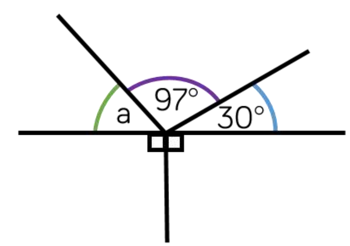
- How many number sentences can you write from the images?





- Calculate the missing angles.



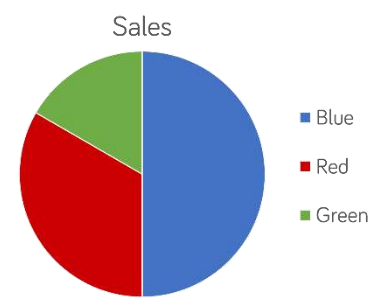


Calculate Angles

Reasoning and Problem Solving

There are five equal angles around a point. What is the size of each angle? Explain how you know.	72° because $360 \div 5 = 72$
Four angles meet at the same point on a straight line. One angle is 81° The other three angles are equal. What size are the other three angles? Draw a diagram to prove your answer.	$180 - 81 = 99°$ $99 \div 3 = 33°$

Here is a pie chart showing the colour of cars sold by a car dealer.



The number of blue cars sold is equal to the total number of red and green cars sold.

The number of red cars sold is twice the number of green cars sold.

Work out the size of the angle for each section of the pie chart.

Blue : 180°
Red : 120°
Green : 60°

Vertically Opposite Angles

Notes and Guidance

Children recognise that vertically opposite angles share a vertex. They realise that they are equal and use practical examples to show this.

They continue to apply their understanding of angles on a straight line and around a point to calculate missing angles.

Mathematical Talk

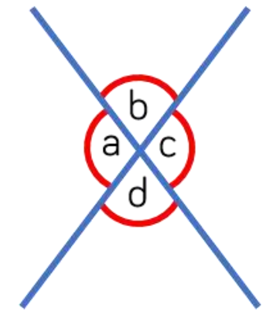
What sentences can we write about vertically opposite angles in relation to other angles?

How can we find the missing angle?

Is there more than one way to find this angle?

Varied Fluency

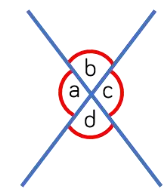
- Take a piece of paper and draw a large 'X'.
  Mark the angles on as shown.
  Measure the angles you have drawn.
  What do you notice about angles b and d?
  What do you notice about angles a and c?
  Is this always the case?
  Investigate with other examples.



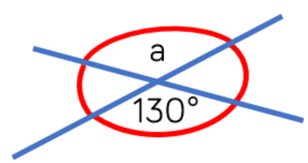
- Use the letters from the diagram to fill in the boxes.

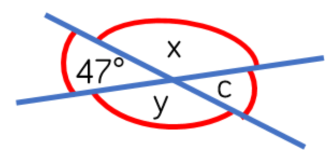
□ = □   □ = □

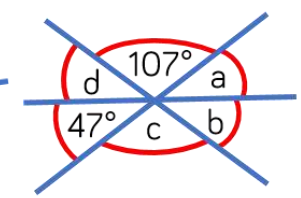
□ + □ = 180°   □ + □ = 180°



- Find the size of the missing angles.





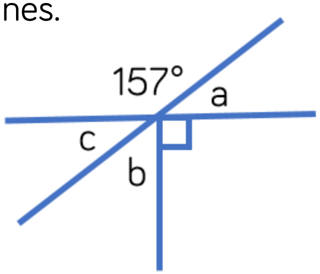


Is there more than one way to find them?

Vertically Opposite Angles

Reasoning and Problem Solving

The diagram below is drawn using three straight lines.



Whitney says that it's not possible to calculate all of the missing angles.

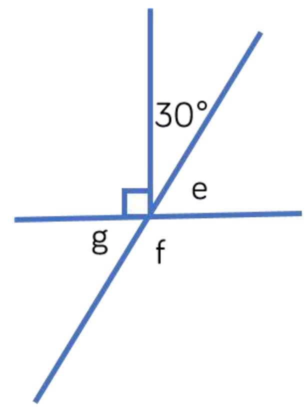
Do you agree? Explain why.

I disagree because:
180 – 157 = 23
so a = 23° because angles on a straight line add up to 180°

Angles a and c are equal because they are vertically opposite so c = 23°

Angles around a point add up to 360° so b = 67°

The diagram below is drawn using three straight lines.



Amir says that angle g is equal to 30° because vertically opposite angles are equal.

Do you agree? Explain your answer.

Find the size of all missing angles.
Is there more than one way to find the size of each angle?

Amir is wrong because g is vertically opposite to e, not to 30° so g would actually be 60°

e = 60°
g = 60°
f = 120°
There are multiple ways to find the size of each angle.

Angles in a Triangle (1)

Notes and Guidance

Children practically explore interior angles of a triangle and understand that the angles will add up to 180 degrees.

Children should apply their understanding that angles at a point on a straight line add up to 180 degrees.

Mathematical Talk

What's the same and what's different about the four types of triangle?

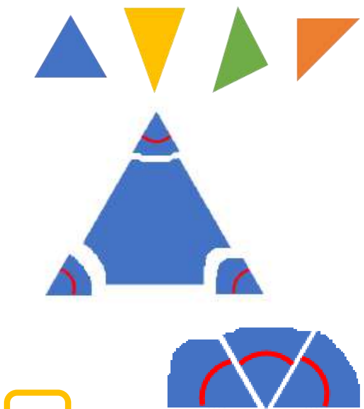
What do the three interior angles add up to? Would this work for all triangles?

Does the type of triangle change anything?

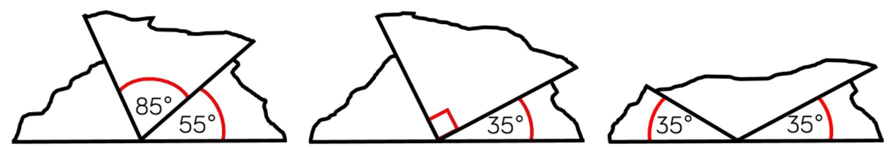
Does the size of the triangle matter?

Varied Fluency

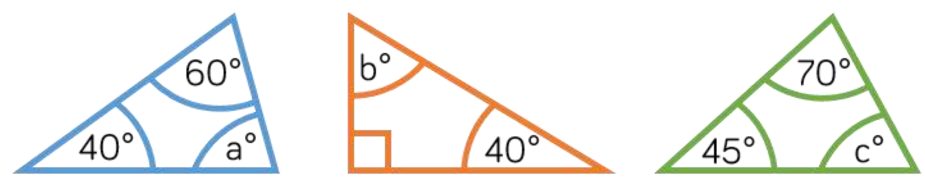
- Use different coloured pieces of card to make an equilateral, isosceles, scalene and right-angled triangle.
  Use a protractor to measure each interior angle, then add them up. What do you notice?
  Now take any of the triangles and tear the corners off. Arrange the corners to make a straight line.
  The interior angles of a triangle add up to ☐



- Calculate the missing angles and state the type of triangle that these corners have been torn from.



- Calculate the missing angles.



Angles in a Triangle (1)

Reasoning and Problem Solving

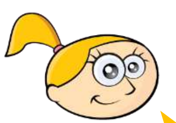
Amir says,

My triangle has two 90° angles.

Can Amir be correct? Can you demonstrate this?

Amir can't be correct because these two angles would add up to 180 degrees, and the third angle can't be 0 degrees.

Eva says,



My triangle is a scalene triangle. One angle is obtuse. One of the angles measures 56°
The obtuse angle is three times the smallest angle.

Work out the size of each of the angles in the triangle.

The interior angles of Eva's triangle are 56°, 93° and 31°

True or False?

A triangle can never have 3 acute angles.

False
Children could use multiple examples to show this.

Angles in a Triangle (2)

Notes and Guidance

Children are introduced to hatch marks for equal lengths. They concentrate on angles in right-angled triangles and isosceles triangles.

Children use their understanding of the properties of triangles to reason about angles.

Mathematical Talk

How can we identify sides which are the same length on a triangle?

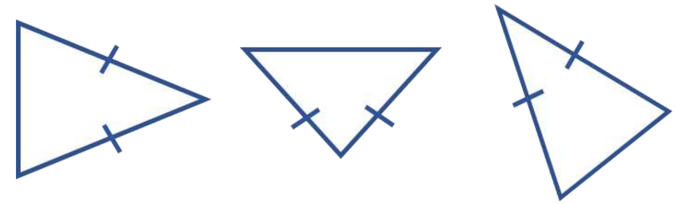
How can we use the use the hatch marks to identify the equal angles?

If you know one angle in an isosceles triangle, what else do you know?

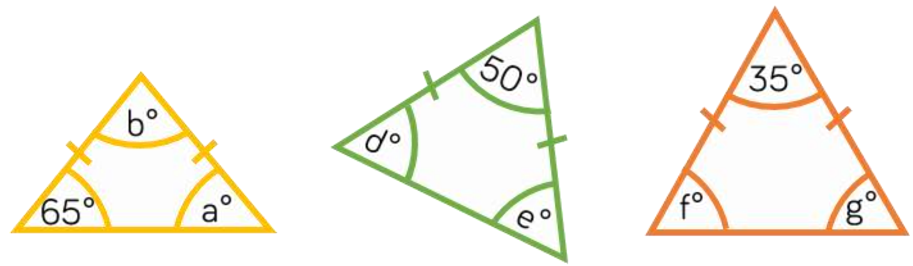
Can you have an isosceles right-angled triangle?

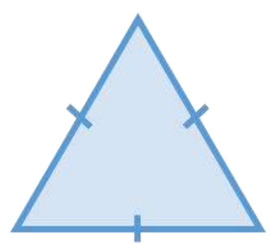
Varied Fluency

- Identify which angles will be identical in the isosceles triangles.



- Calculate the missing angles in the isosceles triangles.



- 

What type of triangle is this?
What will the size of each angle be?
How do you know?
Will this always be the same for this type of triangle?
Explain your answer.

Angles in a Triangle (2)

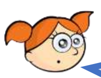
Reasoning and Problem Solving

I have an isosceles triangle.
One angle measures 42 degrees.

What could the other angles measure?

The angles could be:
42°, 42°, 96°
or
42°, 69°, 69°

Alex

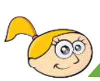


My angles are 70°, 70° and 40°

My angles are 45°, 45° and 90°

Mo

Eva

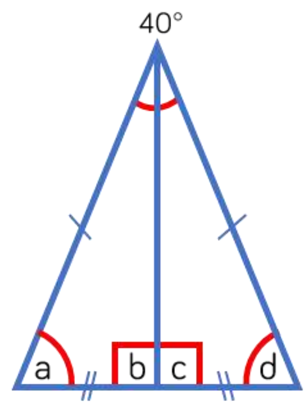


My angles are 60°, 60° and 60°

What type of triangle is each person describing?
Explain how you know.

Alex is describing an isosceles triangle.
Mo is describing an isosceles right-angled triangle.
Eva is describing an equilateral triangle.

How many sentences can you write to express the relationships between the angles in the triangles?
One has been done for you.



40° + a + d = 180°

Possible responses:
20° + a + b = 180°
20° + c + d = 180°
b = 90 °
c = 90 °
b = c
a = d
etc.

Children could also work out the value of each angle.

Angles in a Triangle (3)

Notes and Guidance

Children build on prior learning to make links and recognise key features of specific types of triangle. They think about using this information to solve missing angle problems.

They should also use their knowledge of angles on a straight line, angles around a point and vertically opposite angles.

Mathematical Talk

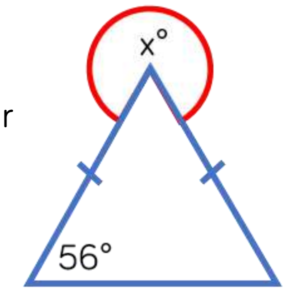
Is it sensible to estimate the angles before calculating them? Are the triangles drawn accurately?

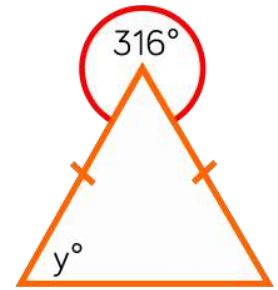
Can you identify the type of triangle? How will this help you calculate the missing angle?

Which angle can you work out first? Why? What else can you work out?

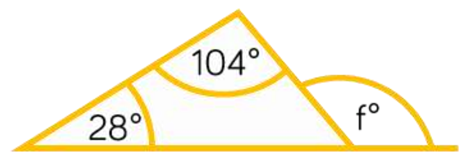
Varied Fluency

- Work out the value of x and y.
  Explain each step of your working.

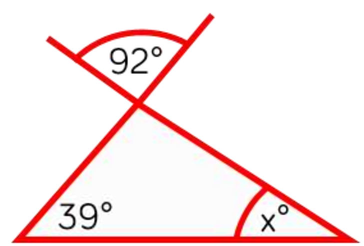


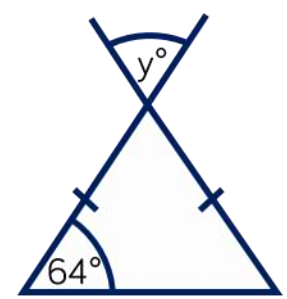


- Work out the value of f and g.
  Explain each step of your working.



- Work out the value of x and y.
  Explain each step of your working.

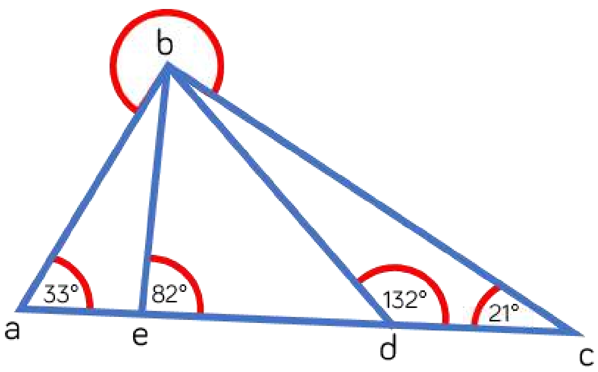
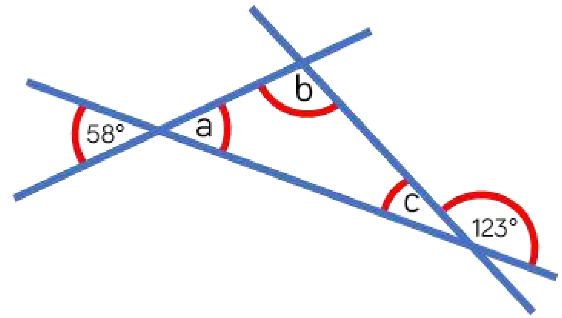




White Rose Maths

Angles in a Triangle (3)

Reasoning and Problem Solving

Calculate the size of the reflex angle b. 	234°
Calculate the size of angles a, b and c.  Give reasons for all of your answers.	a is 58 degrees because vertically opposite angles are equal. c is 57 degrees because angles on a straight line add up to 180 degrees. b is 65 degrees because angles in a triangle add up to 180 degrees.

Angles in Quadrilaterals

Notes and Guidance

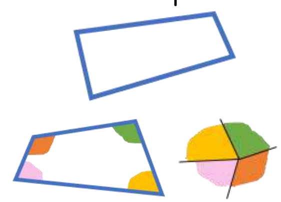
Children use their knowledge of properties of shape to explore interior angles in a parallelogram, rhombus, trapezium etc. They need to learn that angles in any quadrilateral add up to 360°. If they are investigating by measuring, there may be accuracy errors which will be a good discussion point. Children need to have a secure understanding of the relationship between a rectangle, a parallelogram, a square and a rhombus.

Mathematical Talk

Is a rectangle a parallelogram? Is a parallelogram a rectangle?
What do you notice about the opposite angles in a parallelogram?
Is a square a rhombus? Is a rhombus a square?
What do you notice about the opposite angles in a rhombus?
What is the difference between a trapezium and an isosceles trapezium?
If you know 3 of the interior angles, how could you work out the fourth angle?

Varied Fluency

- Take two quadrilaterals.



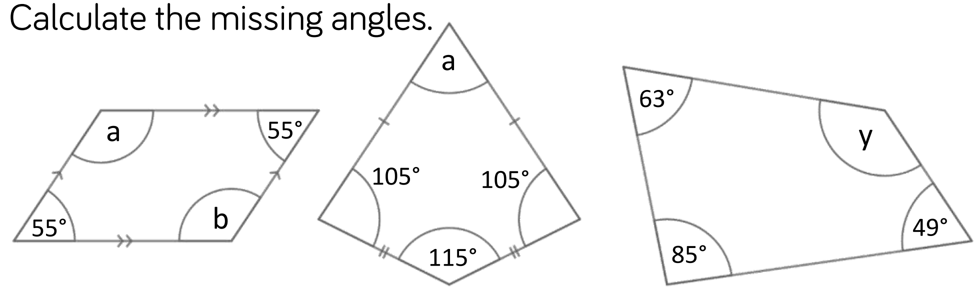
For the first quadrilateral, measure the interior angles using a protractor.
For the second, tear the corners off and place the interior angles at a point as shown.

What's the same? What's different? Is this the case for other quadrilaterals?

- Here are two trapeziums. What's the same? What's different?

Can you draw a different trapezium?
Measure the interior angles of each one and find the total.

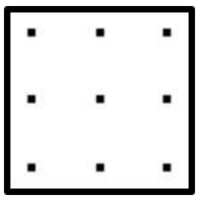
- Calculate the missing angles.



Angles in Quadrilaterals

Reasoning and Problem Solving

How many quadrilaterals can you make on the geoboard?



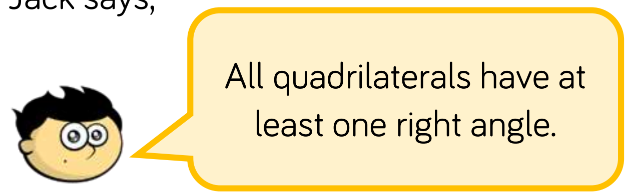
Identify the names of the different quadrilaterals.

What do you notice about the angles in certain quadrilaterals?

If your geoboard was 4 × 4, would you be able to make any different quadrilaterals?

There are lots of different quadrilaterals children could make. They should notice that opposite angles in a parallelogram and rhombus are equal.
They should also identify that a kite has a pair of equal angles, and some kites have a right angle.
On a larger grid, they could draw a trapezium without a right angle.

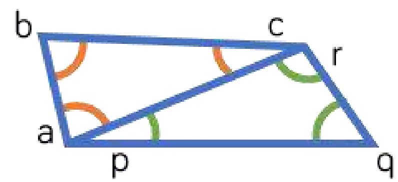
Jack says,



Draw two different shapes to prove Jack wrong. Measure and mark on the angles.

Examples:
Trapezium (without a right angle)
Rhombus
Parallelogram

This quadrilateral is split into two triangles.



Use your knowledge of angles in a triangle to find the sum of angles in a quadrilateral.

Split other quadrilaterals into triangles too. What do you notice?

Children should find that angles in all quadrilaterals will always sum to 360 degrees.

Angles in Polygons

Notes and Guidance

Children use their knowledge of properties of shape to explore interior angles in polygons.

Children explore how they can partition shapes into triangles from a single vertex to work out the sum of the angles in polygons.

They use their knowledge of angles on a straight line summing to 180° to calculate exterior angles.

Mathematical Talk

What is a regular polygon? What is an irregular polygon?

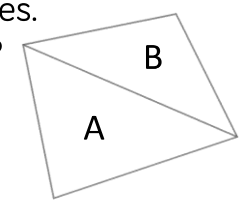
What is the sum of interior angles of a triangle?

How can we use this to work out the interior angles of polygons?

Can we spot a pattern in the table? What predictions can we make?

Varied Fluency

Draw any quadrilateral and partition it into 2 triangles.
What do the interior angles of triangle A add up to?
What do the interior angles of triangle B add up to?
What is the sum of angles in a quadrilateral?



Use the same method to complete the table.

Shape	No. of sides	No. of triangles	180 × no. of triangles	Sum of internal angles
Quadrilateral	4	2	180 × 2	360°
Pentagon	5	3		
Hexagon				
Heptagon				

What do you notice?
Can you predict the angle sum of any other polygons?

Angles in Polygons

Reasoning and Problem Solving

Use the clues to work out what shape each person has.

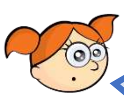
Dora

My polygon is made up of 5 triangles.

The sum of my angles is more than 540° but less than 900°

Tommy

Alex



The sum of my angles is equivalent to the sum of angles in 3 triangles.

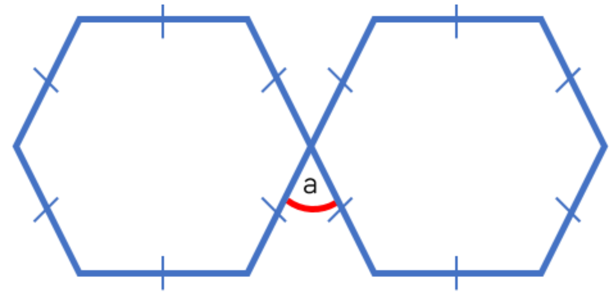
What is the sum of the interior angles of each shape?

Dora:
Heptagon – 900°

Tommy:
Hexagon – 720°

Alex:
Pentagon – 540°

Here are two regular hexagons.



The interior angles of a hexagon sum to 720°
Use this fact to work out angle a in the diagram.

60°

Drawing Shapes Accurately

Notes and Guidance

Children begin by drawing shapes accurately on different grids such as squared and dotted paper. They then move on to using a protractor on plain paper.

Children use their knowledge of properties of shapes and angles, as well as converting between different units of measure.

Mathematical Talk

What do you know about the shapes which will help you draw them?

How can we ensure our measurements are accurate?

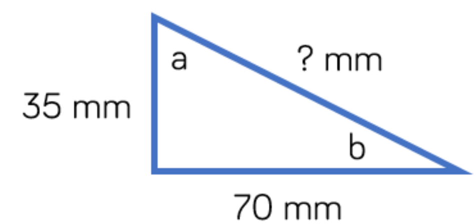
How would you draw a triangle on a plain piece of paper using a protractor?

Varied Fluency

On a piece of squared paper, accurately draw the shapes.

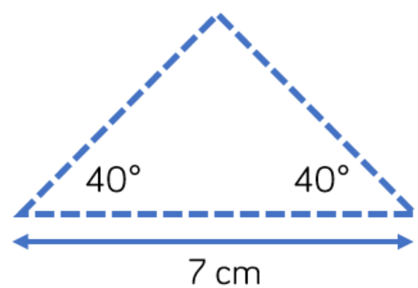
- A square with perimeter 16 cm.
- A rectangle with an area of 20 cm^2.
- A right-angled triangle with a height of 8 cm and a base of 6 cm.
- A parallelogram with sides 3 cm and 5 cm.

Draw the triangle accurately on squared paper to work out the missing length. Measure the size of angles A and B.



Rosie has been asked to draw this triangle on plain paper using a protractor.

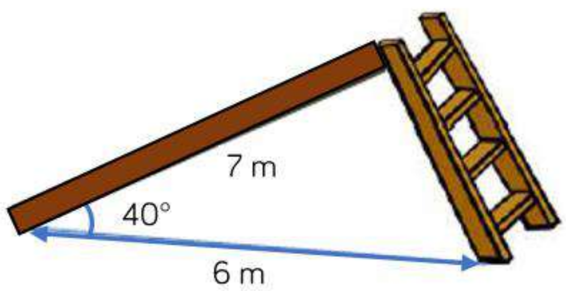
Create a step-by-step plan to show how she would do this.



Drawing Shapes Accurately

Reasoning and Problem Solving

Mr Harrison is designing a slide for the playground.



Use a scale of 1 cm to represent 1 m.

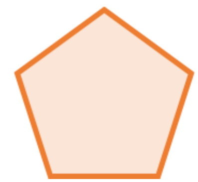
Draw a scale diagram.

Use the diagram to find out how long Mr Harrison needs the ladder to be.

Children will have to use the scale to give their answer in m once they have measured it in cm.

The ladder should be approximately 4.5 m

What is the size of each interior angle of the regular shape below.



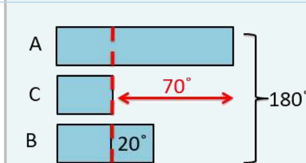
Accurately draw a regular pentagon with side length 5 cm.

108°

Eva has drawn a scalene triangle.
Angle A is the biggest angle.
Angle B is 20° larger than angle C.
Angle C is the smallest angle, and it is 70° smaller than angle A.

Use a bar model to help you calculate the size of each angle, then construct Eva's triangle.

Is there more than one way to construct the triangle?



Angle A: 100°
Angle B: 50°
Angle C: 30°
These angles would work with different side lengths.

Nets of 3-D Shapes

Notes and Guidance

Children use their knowledge of 2-D and 3-D shapes to identify three-dimensional shapes from their nets.

Children need to recognise that a net is a two-dimensional figure that can be folded to create a three-dimensional shape.

They use measuring tools and conventional markings to draw nets of shapes accurately.

Mathematical Talk

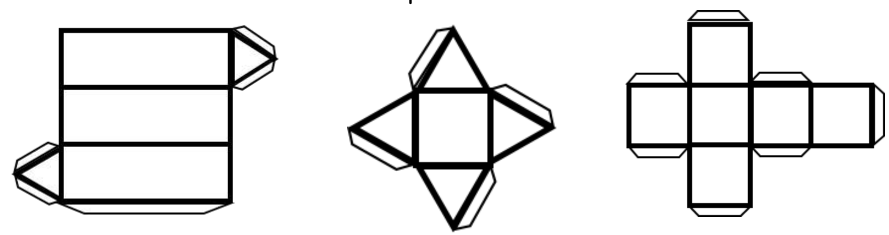
Looking at the faces of a three-dimensional shape, what two-dimensional shapes can you see?

What is a net? What shape will this net make? How do you know? What shape won't it make?

If you make this net, what would happen if you were not accurate with your measuring?

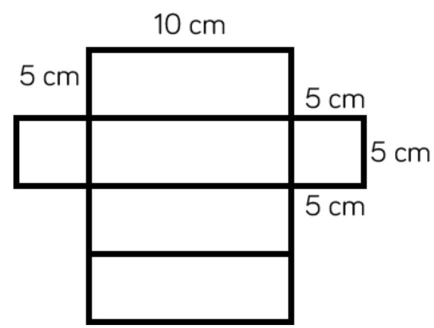
Varied Fluency

- What three-dimensional shape can be made from these nets?



Identify and describe the faces of each shape.

- Accurately draw this net.
Cut, fold and stick to create a cuboid.

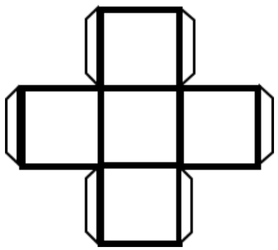


- Draw possible nets of these three-dimensional shapes.

Nets of 3-D Shapes

Reasoning and Problem Solving

Dora thinks that this net will fold to create a cube.



Do you agree with Dora?
Explain your answer.

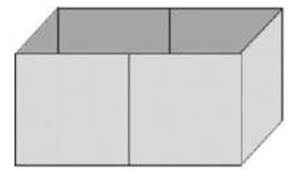
Dora is incorrect because a cube has 6 faces, this net would only have 5

Use Polydron to investigate how many different nets can be made for a cube.

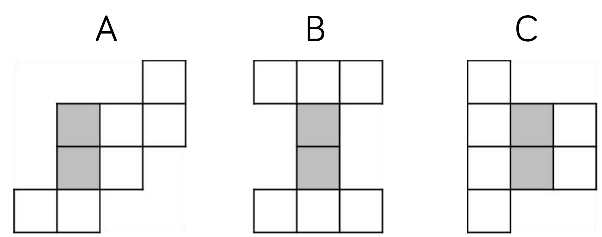
Is there a rule you need to follow?
Can you spot an arrangement that won't work before you build it?
How do you know why it will or won't work?
Can you record your investigation systematically?

There are 11 possible nets.

Here is an open box.



Which of the nets will fold together to make the box?
The grey squares show the base.



B and C